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An Improved Simulated Annealing Approach for Optimal Layout of Agricultural Irrigation Systems

Suiyan Lin*

Business School, Shandong Normal University, Jinan, 250014, China
* **Corresponding author**: Suiyan Lin (Email: lin272720@outlook.com)

Abstract: Efficient allocation of irrigation facilities is crucial for sustainable agricultural water management. Optimizing the layout of irrigation systems under nonlinear cost structures and discrete decision variables, however, remains a challenging combinatorial problem. This study proposes an Improved Simulated Annealing Algorithm (ISA) to optimize the static configuration of an irrigation network consisting of sprinklers, water tanks, and pipelines. The objective is to minimize total construction costs while ensuring complete irrigation coverage. The algorithm incorporates three enhancement mechanisms—memory function, reheating operation, and dual-threshold control—to enhance global search capability and convergence stability. Additionally, a Minimum Spanning Tree (MST) method is embedded to dynamically optimize pipeline topology during iterations. Experimental results on a 1-hectare farm in Changchun, Jilin Province, China, demonstrate that the ISA algorithm achieves an optimal configuration with 15 sprinklers and 10 water tanks, minimizing total construction cost while maintaining full irrigation coverage. The spatial distribution of facilities exhibits an economically rational pattern: sprinklers are concentrated near water sources, while water tanks are placed in remote areas. The proposed ISA provides a robust and scalable approach for irrigation system layout optimization and has potential applications in other agricultural network design problems.

Keywords: Irrigation system optimization; Improved Simulated Annealing; Minimum Spanning Tree; Combinatorial optimization; Cost minimization

1. Introduction

Water scarcity has become one of the most pressing challenges for sustainable agricultural development worldwide. Agriculture consumes approximately 70% of global freshwater resources, with nearly half lost due to inefficient irrigation management. Traditional irrigation systems are often characterized by high energy consumption and low water use efficiency, highlighting the urgent need for intelligent irrigation planning and optimization. In recent years, researchers have applied various mathematical and heuristic optimization methods—such as Genetic Algorithms (GA)[1], Particle Swarm Optimization (PSO)[2][3][4][5], and Simulated Annealing (SA)[6][7]—to design efficient irrigation systems. While these methods have achieved significant progress, several challenges remain unresolved: (1) the nonlinear cost structure of pipelines complicates the optimization process; (2) the discrete nature of equipment placement increases computational complexity; and (3) standard algorithms often exhibit poor convergence stability in large-scale spatial problems.

To address these issues, this paper introduces an Improved Simulated Annealing Algorithm (ISA) that integrates multi-level search mechanisms and adaptive control strategies. The algorithm is designed to minimize total system cost while ensuring full irrigation coverage and rational spatial distribution of facilities. The experiments were conducted based on the measured dataset from a farm in Changchun, Jilin Province, China, and the results demonstrated the effectiveness and scalability of the ISA algorithm in irrigation system layout optimization.

2. Model Formulation

2.1. Problem Description

The optimization problem involves determining the spatial arrangement of sprinklers and water tanks to ensure complete irrigation coverage of the farm while minimizing total construction costs. Each facility covers a circular area of 15 m radius, and the farm (100 m × 100 m) is discretized into a 5 × 5 grid with 25 candidate positions.

2.2. Decision Variables

A binary decision vector is defined, where $s_i = 1$ indicates that a water tank is installed at position i, and $s_i = 0$ denotes a sprinkler.

$$S = (s_1, s_2, \dots, s_{25}) \tag{1}$$

2.3. Objective Function

The total construction cost $C_{total}(S)$ consists of two parts:

$$\min_S C_{total}(S) = C_{pipe}(S) + C_{tank}(S) \tag{2}$$

(1) Tank cost:

Each tank serves its surrounding area and has a fixed unit cost:

$$C_{tank}(S) = 1900 \sum_{i=1}^{25} s_i \tag{3}$$

(2) Pipeline cost:

Pipelines connect all sprinklers to the river via a minimum spanning tree (MST). The cost of each segment e depends on its length L_e and flow rate Q_e:

$$C_{pipe}(S) = \sum_{e \in MST(S)} (50 L_e^{1.2} + 0.1 Q_e^{1.5}) \tag{4}$$

2.4. Constraints

(1) Coverage constraint: Each point in the farm must be within 15 m of at least one sprinkler or tank.

(2) Distance constraint: The minimum spacing between adjacent sprinklers is not less than 15 m.

(3) Topology constraint: All active nodes (sprinklers) must be connected through the MST network.

3. Method

3.1. Data Source

The experiments were conducted based on the measured dataset from a farm in Changchun, Jilin Province, China. The dataset includes spatial coordinates of the farm area, water sources, and candidate positions for sprinklers and water tanks.

3.2. Standard Simulated Annealing

The classical SA algorithm imitates the physical annealing process of solids, where a system gradually cools from a high temperature to a stable low-energy state. At each iteration, a new candidate solution is accepted based on the Metropolis criterion:

$$p = e^{-\Delta C/T_k} \tag{5}$$

where ΔC is the change in cost and T_k is the current temperature. Although SA has global convergence properties, it often suffers from slow convergence and sensitivity to parameter tuning.

3.3. Improvements in ISA

To overcome these limitations, the following enhancements, inspired by the improved simulated annealing algorithm proposed in Guan et al. for the Double-layer Corridor Allocation Problem[8], are incorporated:

(1) Memory mechanism: The algorithm continuously stores the best solution S_{best} found so far, ensuring that global optima are not lost when inferior solutions are temporarily accepted.

(2) Reheating operation: When the search stagnates, the temperature is reset to the initial high level T_0, allowing the algorithm to escape local minima.

(3) Dual-threshold control: Two adaptive thresholds, R_{in} and R_{out}, are introduced to terminate inner and outer loops efficiently. When no improvement is observed for R_{out} iterations, reheating is triggered.

3.4. Algorithm Procedure

(1) Initialization:

Set initial temperature $T_0 = 50000$, cooling rate $\alpha = 0.97$.

Randomly generate an initial feasible solution S_0.

(2) Neighborhood generation:

Flip operation: randomly toggle one equipment type.

Swap operation: randomly exchange two positions.

(3) Acceptance criterion:

If $\Delta C < 0$, accept new solution; otherwise accept with probability $e^{-\Delta C/T_k}$.

(4) Update and reheating:

Update S_{best} when a better solution is found.

Trigger reheating when stagnation exceeds R_{out}.

(5) Termination:

Stop when temperature T< threshold or maximum iteration is reached.

3.5. Complexity Analysis

ISA has a computational complexity of $O(n^2 \log n)$ due to the embedded MST recalculation. Despite this, the heuristic design allows efficient convergence even for large-scale farms.

4. Results and Discussion

4.1. Experimental Setup

A 1-hectare farm (100 m × 100 m) was selected as the test case. The field was divided into a 5×5 grid, yielding 25 candidate positions for placing water tanks or sprinklers. The irrigation radius was set to 15 m, and the grid spacing was 20 m. The algorithm parameters were initialized as follows: initial temperature $T_0 = 50{,}000$, cooling rate $\alpha = 0.97$, inner loop threshold $R_{in} = 50$, and outer loop threshold $R_{out} = 40$. The Improved Simulated Annealing (ISA) algorithm was implemented in Python, with a preset maximum iteration of 1000.

4.2. Convergence Performance

The ISA algorithm converged rapidly, reaching the global optimal solution after only 66 iterations, well before the preset iteration limit. The initial solution contained 9 water tanks, with a total cost of 44,108.79 CNY. At iteration 40, a reheating operation was triggered, resulting in a reduction of the current best cost to 42,486.33 CNY. No further improvement was observed after reheating, causing the algorithm to terminate early. This behavior demonstrates the algorithm's ability to quickly lock onto high-quality solutions in the early stage of the search. The cost history over the 66 iterations (including the initial solution) is shown in Figure 1.

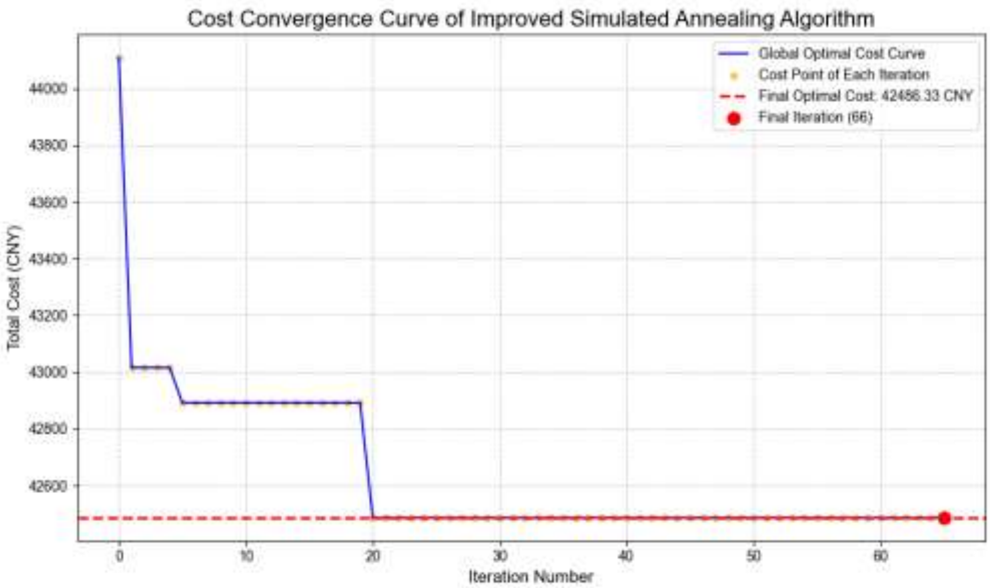

Figure 1 Cost Convergence Curve of Improved Simulated Annealing Algorithm.

4.3. Optimal Layout

The optimal layout obtained by the ISA consists of 10 water tanks and 15 sprinklers. Water tanks are primarily located in remote areas to minimize long-distance water delivery costs, while sprinklers are concentrated near water sources to reduce pipeline length. The specific coordinates of the facilities are as follows:

(1) Water tank positions: (50,10), (70,10), (90,10), (30,30), (70,30), (90,30), (70,50), (50,90), (70,90), (90,90)

(2) Sprinkler positions: (10,10), (30,10), (10,30), (50,30), (10,50), (30,50), (50,50), (90,50), (10,70), (30,70), (50,70), (70,70), (90,70), (10,90), (30,90)

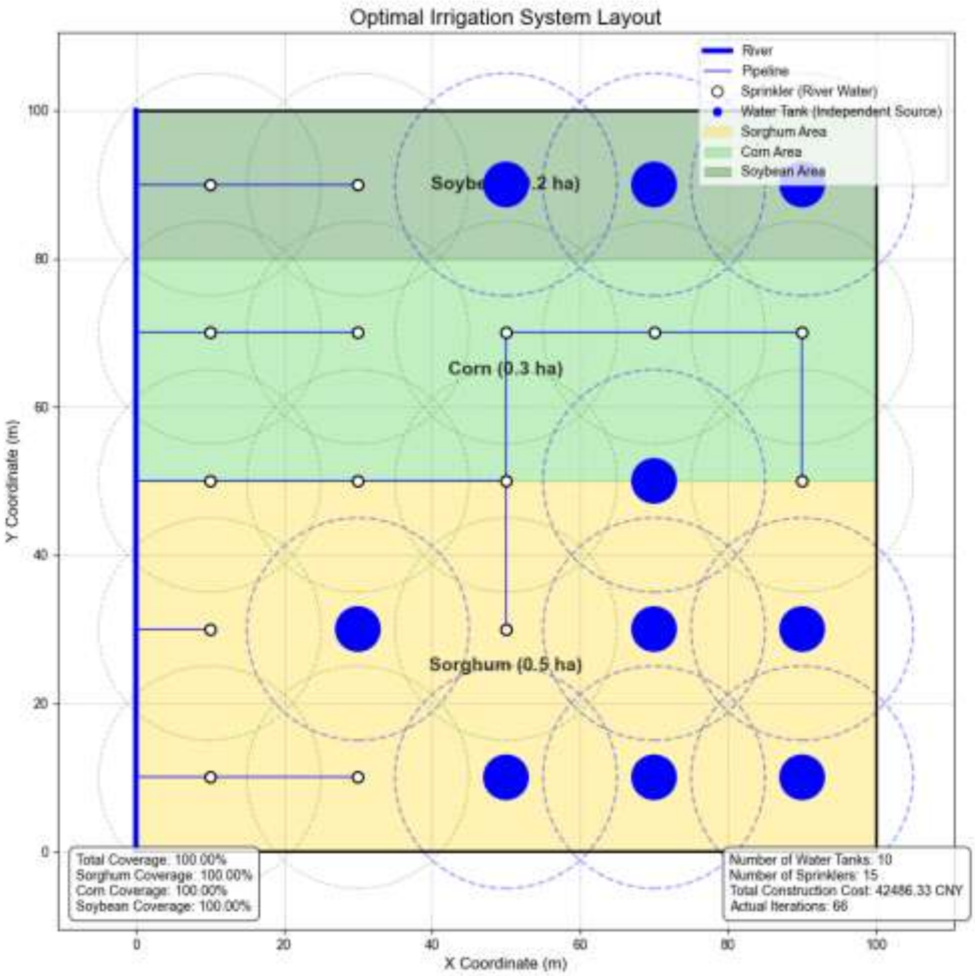

Figure 2 Optimal Irrigation System Layout.

This configuration ensures 100% coverage for all crops, including sorghum, corn, and soybean, fully satisfying the coverage constraints.

4.4. Cost and Coverage Analysis

The total construction cost for the optimal solution is 42,486.33 CNY, representing a reduction of approximately 3,622.46 CNY from the initial solution, indicating the effectiveness of the ISA in cost optimization. The coverage rates are summarized as follows:

(1) Total coverage: 100.00%
(2) Sorghum coverage: 100.00%
(3) Corn coverage: 100.00%
(4) Soybean coverage: 100.00%

These results demonstrate that the ISA can achieve an economically efficient layout while ensuring complete field coverage.

5. Conclusion

This study develops an Improved Simulated Annealing (ISA) algorithm for the optimal layout of irrigation systems, addressing challenges associated with nonlinear cost structures, discrete facility placement, and large-scale spatial complexity. The ISA integrates a memory mechanism, reheating operation, and dual-threshold control, enabling effective global search and stable convergence. The incorporation of a Minimum Spanning Tree (MST) method allows dynamic optimization of pipeline topology, further improving cost efficiency.

Experimental results on a 1-hectare farm in Changchun, Jilin Province, China, demonstrate that the ISA algorithm identifies an optimal configuration of 10 water tanks and 15 sprinklers, reducing the total construction cost to 42,486.33 CNY while achieving full irrigation coverage for all crops. The spatial distribution of facilities reflects economic rationality: sprinklers are concentrated near water sources to minimize pipeline length, whereas water tanks are positioned in remote areas to reduce long-distance water delivery costs.

Overall, the proposed ISA algorithm provides a robust, scalable, and practical approach for irrigation system layout optimization. Its effectiveness in minimizing costs while ensuring complete coverage suggests its potential applicability to broader agricultural network design problems, offering valuable guidance for intelligent water resource management and sustainable agricultural development.

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