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A Review of Research and Applications of Variational Quantum Algorithms

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Abstract: In recent years, as datasets have grown larger, quantum computing has shown potential in addressing the rapidly increasing costs of data processing due to its superposition and entanglement capabilities. Limited by quantum hardware and the number of controllable qubits, variational quantum algorithms (VQAs) are algorithms suitable for NISQ devices, capable of computation on medium-scale noisy devices. VQAs are hybrid classical-quantum algorithms that use parameterized quantum circuits to prepare parameterized quantum states and employ classical optimizers to minimize objective functions, enabling efficient solutions for complex problems. Classical optimizers reduce the stringent requirements on quantum hardware and can achieve quantum advantage on current devices. This paper systematically elaborates on the core foundational theories of quantum computing and neural networks, with a focus on reviewing the research and application status of VQAs in three main scenarios: machine learning (supervised, unsupervised, and reinforcement learning), solving systems of equations (linear and nonlinear), and network attack detection. It analyzes the current technical bottlenecks in this field and provides prospects for future development, offering a reference for further research on variational quantum algorithms.

Keywords: Variational Quantum Algorithms; Quantum Machine Learning; Quantum Neural Networks; Network Attack Detection

1. Introduction

Since the birth of the first electronic computer in 1946, classical computing has achieved exponential growth relying on Moore's Law. Its binary bit-based logic operation architecture has supported modern information technology systems such as artificial intelligence, big data analytics, and complex industries. However, with the continuous development of computer technology and the increase in information data, classical computers are approaching the limits of Moore's Law and face exponentially complex data, making existing computer hardware and costs difficult to manage. Quantum computing offers an effective solution to current problems. Leveraging the uniqueness of quantum mechanics, quantum computing has emerged and has significant advantages in solving problems that are difficult for classical computers. However, traditional quantum computing struggles to isolate noise, and we are currently in the era of Noisy Intermediate-Scale Quantum (NISQ) devices [1]. Variational Quantum Algorithms (VQAs) provide a new solution.

Traditional quantum algorithms, such as Shor's algorithm and Grover's algorithm, require designing complex quantum gate sequences, have high requirements for quantum circuits, require a large number of qubits, and the quantum circuit depth is extremely deep. In contrast, variational quantum algorithms are a type of hybrid quantum-classical algorithm that divides computation into quantum and classical parts. Observables are obtained on a quantum computer by optimizing parameterized functions, and classical optimizers iteratively find the optimal parameter values. Compared to traditional quantum algorithms, variational quantum algorithms have fixed parameterized quantum circuits and lower qubit requirements. Variational quantum algorithms have been applied in various fields, including solving discrete logarithms [2], image classification [3], and classification problems [4]. Compared to classical quantum algorithms, variational quantum algorithms hold a very promising application prospect. Therefore, this paper mainly introduces the fundamentals of quantum algorithms and the applications of variational quantum algorithms in the fields of machine learning and equation solving.

2. Current domestic and international development status

Quantum computing is an important part of quantum information, using features such as superposition and entanglement of qubits to achieve computational power far beyond that of classical computers. Quantum computing capabilities require strong hardware support as well as well-designed and optimized algorithms. Quantum neural networks combine the advantages of quantum computers and neural networks. With the help of quantum neural networks, it is hoped that some problems existing in traditional machine learning can be addressed. This section will introduce the current research status of quantum algorithms, quantum neural networks, and machine learning.

2.1. Current Status of Quantum Algorithm Development

Quantum computing performs calculations by leveraging quantum mechanical principles, offering significant advantages over classical computing in parallel processing and information representation. Its applications span cryptography and information security [4], biomedicine, material science, artificial intelligence, and big data processing. In the modern information age, quantum computing has become an indispensable field, with its applications increasingly widespread.

The origins of quantum mechanics can be traced back to 1900, when Max Planck [5] first proposed the theory, marking the inception of quantum mechanics. Subsequently, Shor's [7]quantum algorithm for integer factorization transformed quantum science into a cutting-edge and prominent research topic. By utilizing quantum superposition and entanglement, this algorithm addresses computational problems that are intractable for classical computers, while also posing a fundamental challenge to the classical RSA encryption standard. The development of Grover's [8] search algorithm further propelled advancements in quantum computing. In 1999, Feng Mang [9] and colleagues in China analyzed the implementation of Grover's algorithm in ion traps, demonstrating a two-qubit Grover search.

In 2009, three scientists from the Massachusetts Institute of Technology (MIT) proposed the HHL algorithm for solving linear systems of equations, achieving exponential speedup compared to classical computing methods. In 2014, Farhi et al. introduced the Quantum Approximate Optimization Algorithm (QAOA) [9], which combines quantum computing properties with classical optimization techniques to tackle combinatorial optimization problems. IBM launched its cloud-based quantum computing service, the Quantum Experience, in 2016, enabling users to access and manipulate quantum computing resources via a web platform. In 2020, China's quantum computer "Jiuzhang" achieved rapid solution of Gaussian Boson Sampling (GBS) problems, showcasing breakthrough capabilities in specific computational tasks.

In recent years, quantum hardware has entered the Noisy Intermediate-Scale Quantum (NISQ) era, prompting a shift in quantum algorithm research toward NISQ-compatible designs. Variational Quantum Algorithms (VQAs) are a key example of this trend. These algorithms iteratively adjust parameters using classical optimizers to find the optimal quantum circuit configuration, primarily targeting combinatorial optimization problems and complementing the Quantum Approximate Optimization Algorithm.

2.2. The Current Development of Quantum Neural Networks

Artificial neural networks rely on the computing power of classical computers, simulate the connections between neurons in the human brain, and achieve learning functions through serial or parallel data processing implemented by classical programming. However, as problems grow increasingly complex and data volume expands continuously, traditional neural networks suffer from insufficient optimization methods and high training costs. In contrast, Quantum Neural Networks (QNNs) are innovative machine learning models or algorithms that integrate quantum computing principles with classical neural networks. By leveraging the characteristics of quantum superposition and entanglement to achieve efficient parallel information processing, they can outperform classical neural networks in computational capability for specific tasks.

Since Professor Kak from the United States proposed the concept of quantum neural computing in 1995[11], scientists from various fields have shown a growing interest in relevant research and put forward numerous related models aimed at enhancing the computing power of quantum neural networks. In 1997, Bonnell et al. optimized a model of classical neural networks, and the results demonstrated that the system possessed dynamically attractive storage characteristics[12]. In 2000, Matsui et al. proposed an improved quantum backpropagation neural network model[13], which replaced classical bits with qubits and realized weight updates by means of quantum gates. Subsequently, Li proposed a quantum parallel self-organizing model that can achieve autonomous organization and mapping in a quantum computing environment, and it was proven that this model had a faster convergence rate than artificial neural networks[14].

In 2019, Wright et al. presented a parameterized quantum state neural network[15]. In recent years, research on quantum neural networks has become increasingly vibrant, and they have exhibited tremendous research potential in fields such as function approximation and convolutional neural networks.

2.3. The Current Development of Quantum Machine Learning

In recent years, an increasing number of products based on machine learning models have been applied in daily life. The performance of machine learning algorithms is related to the scale of datasets. With the continuous growth of global data, the scale of datasets has been expanding significantly. Traditional machine learning algorithms can no longer meet the practical demands, while quantum machine learning has demonstrated outstanding performance in this context.

The initial quantum supremacy experiment conducted by Google[16] has already proven the advantages of Noisy Intermediate - Scale Quantum (NISQ) devices over traditional ones. Subsequently, Yoo et al. conducted research on binary classification problems in both the quantum machine learning and classical machine learning fields, and found that the efficiency of quantum machine learning is significantly higher than that of classical machine learning[17].

In 2015, Professor Long Guilu and his colleagues published a comprehensive review on the application fields of quantum machine learning and big data analytics[18]. In the same year, Li et al. also verified the quantum - enhanced support vector machine algorithm through experiments[19].

Currently, the "Zuchongzhi 2.1" quantum processor developed by Zhu et al. from the University of Science and Technology of China has been proven in experiments to be 500 times more efficient than the Sycamore processor, which fully demonstrates quantum computational advantage.

At present, restricted by quantum hardware, there are several critical challenges to be addressed, such as the insufficient number of qubits and the noise problem generated by quantum circuits. Nevertheless, variational quantum algorithms hold great potential in achieving quantum advantage and have become the primary algorithms for gaining quantum advantage in the field of machine learning.

3. Related Concepts

3.1. Quantum Computing

He essence of quantum superposition is "uncertainty", which is completely different from the "certainty" of physical states in classical physics. A classical bit can only be in one of two states, 0 or 1, while a quantum bit (qubit) can exist in multiple states, and it is not a switch between different states. A qubit can be in a superposition state of $|0\rangle$ and $|1\rangle$ simultaneously, which can be mathematically expressed by the following formula, where α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$. It can represent any continuous state between $|0\rangle$ and $|1\rangle$. The most classic example of quantum superposition is "Schrödinger's Cat".

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

Quantum Entanglement: It refers to an inseparable correlated state formed by two or more particles after interaction. Even if they are far apart, measuring the state of one particle will instantly cause a change in the state of another particle.

Quantum Coherence and Decoherence: Quantum coherence is the "correlation" of quantum states in a quantum system, including properties such as interference, diffraction, and entanglement. Quantum decoherence is a process in which a quantum system loses coherence and the characteristics of superposition states due to environmental interaction, transitioning from a quantum state to a classical state.

Classical Bits vs. Qubits: In a classical computer, a classical bit can be in a state represented by bit 0 or bit 1. In a quantum computer, a qubit is a two-level quantum system, with two possible states denoted by $|0\rangle$ and $|1\rangle$. The state of a qubit can be represented by a two-dimensional vector.

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix},\quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \tag{2}$$

Due to the principle of superposition, any quantum state of this system can be expressed by Formula , where α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$.

The measurement result shows that α^2 represents the probability of the $|0\rangle$ state, and β^2 represents the probability of the $|1\rangle$ state. That is to say, unlike the state of a classical bit, a qubit is not in a certain state of $|0\rangle$ and $|1\rangle$, but in a linear combination state of them. According to the normalization of coefficients, a general quantum state can also be represented by a Bloch sphere.

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\frac{\theta}{2}|1\rangle = \begin{bmatrix}\cos\frac{\theta}{2}\\ e^{i\varphi}\frac{\theta}{2}\end{bmatrix} \tag{3}$$

Among them $0 \leqslant \theta \leqslant \pi,\ 0 \leqslant \varphi \leqslant 2\pi$.

Similar to classical computers, quantum computers can manipulate quantum states through the operation of quantum gates. Any single-qubit gate can be represented by a 2x2 matrix and has four parameters.

$$U = \exp(i\gamma) = \begin{bmatrix}\exp(i\phi)\cos\theta & \exp(i\beta)\sin\theta\\ -\exp(-i\beta)\sin\theta & \exp(-i\phi)\cos\theta\end{bmatrix} \tag{4}$$

Expanding the matrix and simplifying using unitarity gives

$$U_X(\alpha) = \exp(i\alpha\sigma_X) = \begin{bmatrix}\cos\alpha & i\sin\alpha\\ i\sin\alpha & \cos\alpha\end{bmatrix} \tag{5}$$

Of course, Ux and Uy correspond to the operations of rotating a qubit around the Y - axis and Z - axis, respectively. Arbitrary single - qubit gates can be constructed.

Similar to classical circuits, a quantum circuit is composed of various quantum logic gates. Each quantum logic gate corresponds to a unitary operator, which performs the corresponding unitary transformation. The quantum circuit of a common single - qubit gate is shown in Figure 1.

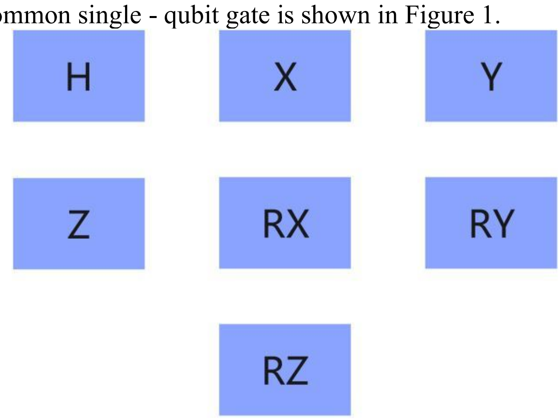

Figure 1 Single-Qubit Gate

Common two-qubit gates include the quantum circuit for the CNOT (controlled-NOT) gate shown in Figure (a), the quantum circuit for the CZ (controlled-Z) gate shown in Figure (b), and the quantum circuit for the SWAP gate shown in Figure (c). Of course, it is composed of three CNOTgates as shown in Figure

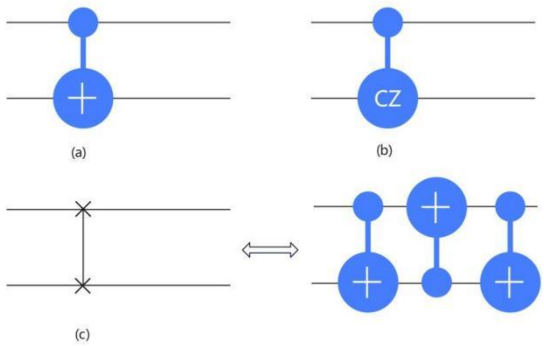

Figure 2 Two-Qubit Gate

The quantum circuit for quantum measurement is shown in Figure 3:

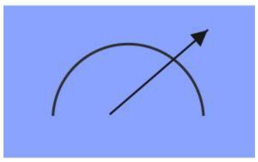

Figure 3 Quantum Measurement Circuit

Unlike classical measurement, quantum measurement changes the state of the system of variables, and the results follow a probability distribution. If the quantum system before measurement is in the state $|\varphi\rangle$, and $\{M_i\}$ is the set of pre-measurement operators, then the probability of the result occurring is:

$$P(i) = \langle\varphi|M_i^{\dagger}M_i|\varphi\rangle \tag{6}$$

Then the state after measurement becomes:

$$\frac{M_i|\varphi\rangle}{\sqrt{\langle\varphi|M_i^\dagger M_i|\varphi\rangle}} \tag{7}$$

Because of $1=\sum_i p(i)=\sum_i \langle\varphi|M_i^\dagger M_i|\varphi\rangle$, So the measurement operator should have $\sum_i M_i^\dagger M_i = I$, Therefore, this equation is complete.

3.2. Neural Networks

An Artificial Neural Network (ANN) is a machine learning model that simulates the structure and function of biological neural networks, referred to as a neural network for short. Designed to mimic the operational mode of biological neural networks, an ANN is a computing system that performs various tasks on large volumes of data [21]. It processes data by simulating the information-handling process of the human brain, which makes neural networks analogous to biological neural networks in nature.

A neural network consists of layers of nodes and an output layer, where the node layers include an input layer and multiple hidden layers in between. Nodes are interconnected via neurons, with corresponding weights and thresholds assigned to these connections. If the output of a node exceeds its threshold, the node will be activated and transmit data accordingly.

The core characteristics of neural networks include distributed storage, parallel processing, and adaptive learning capability. Essentially, a neural network fits complex functions through multi-layer nonlinear transformations, simulating the three components of a neuron: input, integration, and output. Its mathematical expression is as follows:

$$y=f\left(\sum{}_{i=1}^{n} w_i x_i + b\right) \tag{8}$$

4. Machine Learning Based on Variational Quantum Algorithms

In recent years, with the continuous growth of global user data scale and the ever-expanding size of datasets, traditional machine learning algorithms have been struggling to keep pace [22]. Compared with classical machine learning, quantum machine learning can leverage its unique physical properties—such as entanglement, quantum state superposition, and interference—when dealing with large-scale datasets, thus breaking through the bottlenecks that restrict classical machine learning in processing massive volumes of data. Quantum machine learning transcends the computational complexity boundaries of classical machine learning, establishing a novel learning framework rooted in the principles of quantum mechanics. Constrained by the current state of quantum hardware, the limited number of controllable qubits and quantum decoherence are two major challenges facing quantum machine learning. Consequently, achieving quantum advantage on Noisy Intermediate-Scale Quantum (NISQ) devices has become a primary research objective.

Variational quantum algorithm-based machine learning is a branch of quantum machine learning that is tailored to current noisy NISQ devices, enabling the realization of quantum advantage across various machine learning tasks. Leveraging a hybrid classical-quantum architecture, it can address the limitations exhibited by conventional machine learning methods when handling high-dimensional datasets. Its core logic lies in iteratively optimizing the parameters of quantum circuits via classical optimizers to achieve machine learning objectives.

As illustrated in Figure 3.1, the variational quantum algorithm model based on quantum gate circuits employs a quantum circuit computer. Classical data is mapped to quantum states through data feature mapping methods. Parametrized quantum circuits are then used to process the quantum state data. The quantum states are measured and collapsed into classical data. Statistical analysis is performed on the measurement results, which provide feedback for the subsequent classical optimizer. Finally, the classical optimizer iterates repeatedly until the loss function converges.

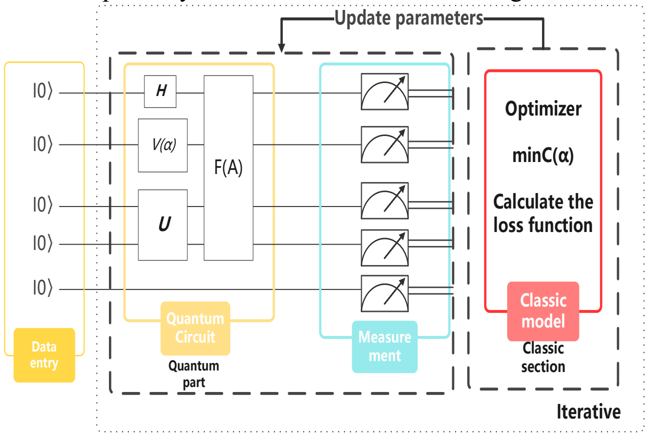

Figure 3.1 Flow Chart of Machine Learning Algorithm Based on VQA

4.1. Supervised Learning

Classical supervised machine learning mainly focuses on classification and regression tasks. In quantum supervised learning, variational quantum algorithms are importantly applied to classification tasks, including quantum convolutional neural networks and shadow quantum circuits. Quantum convolutional neural networks mimic the architecture of classical convolutional neural networks, realizing feature extraction and feature mapping via quantum gates, and they exhibit excellent noise resilience. Since Henderson et al. [22] proposed the quantized convolutional neural network, Liu et al. [24] combined quantized convolutional neural networks with quantum convolutional neural networks to present the quantum-classical convolutional neural network. Compared with classical convolutional neural networks, this method requires a number of qubits that is only related to the size of the feature map, thus demonstrating greater advantages.

Variational shadow quantum circuits integrate local feature extraction with classical neural networks. They show characteristics of high precision and strong robustness in the measurement of noisy quantum states, and can avoid the barren plateau problem, making them suitable for NISQ devices. In image classification tasks, by encoding image pixel features into quantum states, quantum convolutional neural networks can leverage their parallelism to rapidly extract features and improve classification efficiency.

4.2. Unsupervised Learning

In unsupervised learning, variational quantum algorithms are mainly applied to probability distribution modeling and quantum state processing. The Quantum Born Machine (QBM) generates quantum probability distributions for target data via parametrized quantum circuits, enabling the mining of logarithmic features. The Quantum Autoencoder (QAE) is capable of compressing and reconstructing quantum states: it maps high-dimensional quantum states to a low-dimensional space through an encoding circuit and then reconstructs the original quantum states via a decoding circuit, thus achieving high fidelity in quantum data dimensionality reduction tasks. These models demonstrate unique advantages when handling complex probability distributions that are difficult to model with classical unsupervised learning methods.

4.3. Reinforcement Learning

Reinforcement learning is a learning paradigm where an intelligent agent interacts with the environment to find an optimal strategy, driven by predefined reward mechanisms. Classical reinforcement learning typically adopts decision tree models, whereas quantum reinforcement learning replaces neural networks with variational quantum algorithms. Traditional reinforcement learning often uses value function-based approaches, and quantum reinforcement learning can be implemented by leveraging variational quantum algorithms. Chen et al. [25] pioneered the proposal of quantum reinforcement learning based on variational quantum algorithms, among which the Variational Quantum Deep Q-Network (VQ-DQN) uses parametrized quantum circuits to approximate the Q-value function instead of classical neural networks. Its core process is as follows: first, the environmental state is mapped to a quantum state through feature mapping; then, the Q-value is calculated via a parametrized quantum circuit; finally, a classical optimizer is used to minimize the discrepancy between the predicted value and the target value. Nevertheless, this algorithm may suffer from the problem of strategy degradation, which can be addressed by improving the variational quantum circuit. When dealing with reinforcement learning tasks in high-dimensional state spaces, this algorithm can leverage quantum parallelism to achieve computational acceleration. Additionally, it has a small number of parameters, making it suitable for deployment on NISQ devices.

5. Solving Systems of Equations Based on Variational Quantum Algorithms

When tackling high-dimensional and nonlinear systems of equations, traditional classical algorithms are often constrained by bottlenecks such as high computational complexity and low solution efficiency. As one of the core algorithms tailored for Noisy Intermediate-Scale Quantum (NISQ) computers, variational quantum algorithms offer a brand-new pathway for solving systems of equations, thanks to their low quantum resource requirements and hybrid quantum-classical optimization properties.

5.1. Solving Linear Systems of Equations

Currently, the solution of linear systems of equations based on variational quantum algorithms has been applied in power flow calculations [26]. Its core process is as follows: first, the problem is transformed and dimensionality-reduced. For large-scale linear systems of equations, the classical GMRES algorithm can be used for dimensionality reduction to adapt to NISQ devices. Second, the coefficient matrix of the reduced-dimension system of equations is decomposed into a linear combination of several unitary matrices, encoded via a parameterized quantum circuit, and the Hamiltonian of the problem is constructed accordingly. Finally, a classical optimizer is employed to iteratively adjust the parameters of the quantum circuit, driving the Hamiltonian toward the ground state until the loss function converges. The quantum state is then measured, and the final classical data is output.

5.2. Solving Nonlinear Systems of Equations

Nonlinear systems of equations involve nonlinear terms such as variable products and exponents, making them significantly more difficult to solve. Yang Yihang et al. [27] have validated the solution of nonlinear systems of equations based on variational quantum algorithms on the IBM Quantum Cloud Platform. The basic process of solving equations with variational quantum algorithms is as follows: first, optimize the system of equations; then construct the Hamiltonian of the problem based on the optimized system; encode the problem Hamiltonian into quantum states; build a logical quantum circuit using variational quantum algorithms; repeatedly measure the ground state with a classical optimizer to iteratively optimize parameters; perform expectation estimation after each iteration until the loss function converges to a preset threshold.

5.3. Network Attack Detection Based on Variational Quantum Algorithms

Traditional network attack detection systems face two core challenges: limited detection accuracy (especially the recognition rate of zero-day attacks is less than 60%), and model bias caused by data imbalance. Variational quantum circuits demonstrate robustness in addressing component noise issues, thus emerging as one of the most efficient quantum algorithms. However, when the number of quantum circuits is limited, Variational Quantum Neural Networks (VQNNs) exhibit the drawback of low model accuracy. To tackle this problem, the deep variational quantum neural network and Variational Quantum Convolutional Neural Network (VQCNN) proposed by Guan Weiqi [28] have both been validated in network attack detection scenarios.

The deep VQNN model can address the insufficient detection accuracy of shallow quantum circuits. Its core structure consists of a quantum state encoding circuit, multiple layers of variational quantum circuits, and a measurement module. The model enhances feature representation capability by increasing the depth of quantum circuits (usually around 8 layers), and adopts a variational encoding scheme to map classical traffic features (such as protocol type, data byte count, connection count, etc.) into quantum states. This model has been deployed and validated on the IBM Quantum Cloud Platform. Even in the noisy environment of real quantum hardware, it still maintains better detection performance than classical models such as Support Vector Machines (SVM) and Naive Bayes (NB).

The VQCNN Model Adopts a Structure of "One Round of Convolutional Pooling + Deep VQNN" Quantum convolutional filters extract local features through translation-invariant operations, while quantum pooling layers achieve feature dimensionality reduction and retain key information. The subsequent VQNN layers complete the final classification task, effectively balancing detection accuracy and training efficiency.

6. Challenges and Outlook

As a core bridge connecting quantum computing and classical artificial intelligence, variational quantum algorithms demonstrate significant research potential and application value in fields such as machine learning, system of equations solving, and network attack detection. Despite substantial progress made in both theoretical and experimental aspects, variational quantum algorithms still face several key drawbacks and challenges due to the constraints of current NISQ hardware levels and the inherent characteristics of the algorithms themselves. As the depth of quantum circuits increases, the variance of the loss function decays exponentially, flattening the optimization results. Classical optimizers then fall into local optima, leading to the barren plateau phenomenon. In addition, the limited number of controllable qubits and the inability to guarantee the fidelity of quantum gates make it difficult to process high-dimensional network traffic features. Moreover, the growing depth of quantum circuits brings about increased noise, which may even result in quantum computers underperforming classical computers. Therefore, addressing noise issues remains a critical technical challenge at present.

In recent years, with the continuous research on quantum computing conducted by companies such as IBM and Google, the field has maintained a strong development momentum, providing new ideas for solving complex problems across multiple domains and exhibiting great potential in machine learning. Although quantum computing, relying on a limited number of controllable qubits and shallow quantum circuits, can outperform optimal classical algorithms in certain fields, building a universal large-scale quantum computer remains difficult to achieve. Quantum computing is transitioning from theory to practical application and is currently in the stage of algorithm design and hardware adaptation. Nevertheless, in the long run, once fault-tolerant quantum computing is realized, quantum computing is bound to bring revolutionary breakthroughs in fields such as cryptanalysis, drug discovery, artificial intelligence, robotics, and autonomous driving.

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