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Study on Slag Discharge Characteristics of Slurry Discharge Pipe of Slurry Shield

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Abstract: Based on the two-way coupling method of Computational Fluid Dynamics (CFD) and Discrete Element Method (DEM), this paper simulates the transportation process under the interaction of slurry and stone particles in slurry pipeline under different parameter variables. According to the evolution of particle phase and fluid phase with time, the transportation characteristics of slurry pipeline are analyzed. The results show that in the horizontal inclined pipeline system, the velocity of the flow field is centripetally gathered when the slurry flows through the elbow area, and the non-uniformity increases with the increase of the inclination angle. The larger the horizontal inclination angle of the pipeline is, the more significant the mud pressure loss is, and the turbulent kinetic energy is positively correlated with the inclination angle. The accumulation of stone particles is mainly concentrated in the second elbow area of the pipeline. The number of particles increases significantly with the increase of inclination angle, while the average velocity fluctuates less. The results of this study can provide a theoretical basis for the optimization of pipeline laying parameters and anti-clogging design of slurry circulation system of slurry shield.

Keywords: Horizontal inclined pipeline; Inclination angle; Fluid-solid coupling; Slurry shield; Pipeline slag discharge.

1. Introduction

With the continuous progress of science and technology, especially in the field of material science and mechanical engineering, shield technology is gradually evolving towards higher efficiency and safety [1,2]. At the same time, with the vigorous development of tunnel construction, the number of urban underground space development and cross-mountain and cross-sea tunnel projects in China has increased significantly in recent years. The slurry circulation system is a key component of the slurry shield machine. In the process of pipeline transportation, if the slurry and slag particles flow poorly, it is easy to cause pipeline siltation and blockage, resulting in a series of chain reactions such as a sharp increase in energy consumption of the slurry pump and a significant decrease in tunneling efficiency. Therefore, it is of great significance to solve the problem of pipeline blockage and improve the efficiency of pipeline transportation by deeply studying the transportation characteristics of slurry and stone slag particles in the slurry circulation system of slurry shield and the condition of pipeline slag discharge.

In view of the transportation characteristics of slurry and slag particles in the slurry circulation system of slurry shield, domestic and foreign scholars have carried out a lot of research. In order to explore the influence of slurry rheological model on the calculation of pressure loss of shield pipeline, Liu et al. [3] used CFD-DEM coupling method to construct a three-dimensional transient slurry-slurry multiphase flow model, revealed the pressure loss characteristics of slurry shield discharge pipe, and determined the rheological parameters in the model through rheological experiments. Sun et al. [4] found that the initial velocity of pebbles decreases in the order of ellipsoid > flat > near-spherical under the same particle size and pipe inclination. Sun et al. [6] also proposed a multi-objective optimization design method for the structural parameters of the spiral pipe in the slurry shield circulation system. The feasibility of the optimization design was verified by practical engineering cases, and the influence of the main structural parameters on the slurry transportation performance was analyzed. The study of Li et al. [5] showed that under the premise of the same particle size distribution, pebble volume fraction and pipeline inclination, the pressure loss of the pipeline increased exponentially with the increase of slurry flow rate. Xu et al. [7] found that the valve opening, fluid density and pump speed will have a certain impact on the flow and pressure distribution of the slurry pipeline system. The study of Jia et al. [8] showed that the higher the mud density, the higher the transport efficiency in the inclined straight pipe.

In the study of the slag discharge characteristics of the slurry circulation system of the slurry shield, most of the previous scholars focused on the study of single pipeline layout such as horizontal, vertical and inclined, and there were few studies on the slag discharge characteristics of continuous pipelines and the transportation and distribution characteristics of slurry and stone particles. In view of this situation, based on the CFD-DEM coupling calculation method, combined with the characteristics of continuous pipelines, this paper selects horizontal inclined slurry discharge pipelines with different inclination angles in the circulation system, and analyzes their slag discharge characteristics from the aspects of slurry flow field velocity distribution, slurry flow field pressure distribution, stone particle velocity distribution, stone particle position distribution, etc., so as to obtain the transportation law of slurry discharge pipelines under different parameters. Use italic for emphasizing a word or phrase. Do not use boldface typing or capital letters except for section headings (cf. remarks on section headings, below).

2. Numerical theoretical model

The CFD-DEM method solves the characteristics of continuous fluid phase flow field and tracks the trajectory of discrete particle phase by coupling computational fluid dynamics (CFD) and discrete element method (DEM), and exchanges the mass, momentum and energy interaction data between the two phases in real time. This study is based on the Fluent-EDEM coupling platform to achieve the numerical simulation.

2.1. The fluid part-continuous phase

The mathematical models corresponding to the slurry as a fluid in the slurry discharge pipeline of the slurry shield include the continuous equation, the Navier-Stokes equation and the energy conservation equation, which are expressed as:

$$\frac{\partial \alpha_f \rho_f}{\partial t} + \nabla \cdot (\alpha_f \rho_f \boldsymbol{U}_f) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_f \rho_f \boldsymbol{U}_f) + \nabla \cdot (\alpha_f \rho_f \boldsymbol{U}_f \boldsymbol{U}_f) = -\nabla p_f + \nabla \cdot \alpha_f \boldsymbol{\tau} + \alpha_f \rho_f \mathbf{g} + \boldsymbol{F}_{pf} \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_f \rho_f T) + \nabla \cdot (\alpha_f \rho_f \boldsymbol{U}_f T) = \nabla \cdot \left(\frac{k_T}{c_P} \nabla T \right) + s_T \tag{3}$$

In the above formula, αf is the fluid volume fraction; αf is the fluid density; Uf is the fluid velocity; pf is fluid pressure; τ is the viscous stress tensor; g is the acceleration of gravity; Fpf is the force exerted by all particles on the fluid; T is temperature; kt is the heat transfer coefficient; cp is the specific heat capacity of the fluid at constant pressure; sT is a viscous dissipation term. Due to the large diameter and high flow rate of the slurry pipe, it is judged that the slurry flow in the slurry shield pipe is typical turbulence. In this study, the turbulence model of the standard k-epsilon wall function is used.

2.2. Fluid part-discrete phase

Stone particles are used as discrete phases to calculate particle translation and rotation using the Lagrangian method. The momentum conservation and angular momentum conservation equations are shown in (4) and (5), respectively:

$$m_p \frac{d\boldsymbol{v}_p}{dt} = m_p \mathbf{g} + \sum \boldsymbol{F}_c + \boldsymbol{F}_d + \boldsymbol{F}_{Mf} + \boldsymbol{F}_m + \boldsymbol{F}_p \tag{4}$$

In formula (4), mp is the mass of the particle; vp is the particle velocity; Fc is the contact force between particles and between particles and wall; Fd is fluid resistance; FMf is Magnus force; Fm is virtual mass force; Fp is the pressure gradient force.

$$\boldsymbol{I} \frac{dw_p}{dt} = \sum \boldsymbol{T}_c + \boldsymbol{T}_f \tag{5}$$

Where (5) I is the moment of inertia of the particle; wp is the particle angular velocity; Tc is the contact torque; Tf is the torque caused by fluid. The flow dynamics in the two-phase flow is complex, and the particles may be affected by various forces. In this study, the contact force, torque and viscoelastic damping force between the stone and the pipe wall and other stone particles conform to the Hertz-Mindlin non-slip contact model.

3. Simulation validity verification and model establishment

3.1. CFD-DEM coupling process

CFD-DEM coupling is a two-way coupling calculation method based on Euler-Lagrange method. In the simulation model of this paper, the slurry as a continuous phase is initialized by the CFD solver to obtain parameters such as fluid velocity and pressure. Stone particles are used as discrete phases to obtain parameters such as particle velocity and position by initialization. The flow field information of a time step is calculated by the CFD solver, and then the DEM solver is started to iterate at the same time. The particle information is passed to the CFD solver by the coupling interface to calculate the interaction between the particle and the fluid. The effect of the fluid on the particles will be transmitted to the DEM solver through the interface, as the particle volume force affects the movement of the particles, and the effect on the fluid acts on the fluid through the momentum source term. Step by step cycle iteration to achieve the whole process of transient simulation.

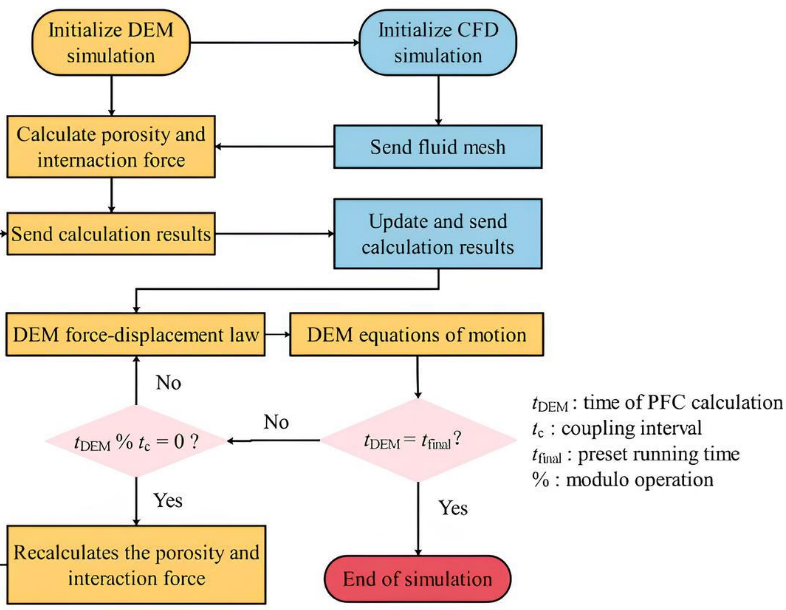

Fig 1. CFD-DEM coupling process

3.2. The establishment of slurry pipe model

3.2.1. Fluid domain mesh model division

As shown in Figure 2, the fluid domain grid model is established. The inner diameter of the pipeline D is 300 mm, the length of the front and rear two straight pipes is 4 000 mm, the angle at the two elbows is θ, the turning radius R is 2 times of D, and the length of the inclined straight pipe in the middle section is 2 000 mm. In the CFD-DEM coupling simulation, in order to improve the calculation accuracy, the grid division method uses hexahedral division, and the grids at the two elbows of the pipeline and the near wall are refined.

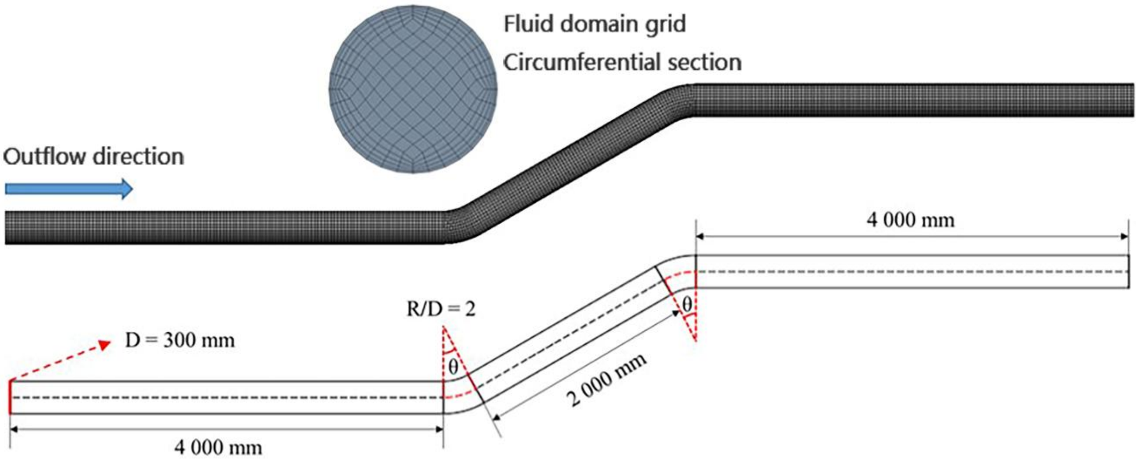

Fig 2. Grid division of pipeline computational domain

3.2.2. Simulation project design

In the CFD model, the inlet boundary is set as the velocity inlet, and the outlet boundary is set as the free outflow. The tube wall is the wall boundary, and it is set as ' static wall ' and ' anti-slip wall '. The negative direction of the Z-axis is defined as the gravity direction, and the gravity acceleration is set to 9.81 m / s2. Both the inlet and outlet adopt the escape condition, and the wall adopts the reflection condition. The SIMPLE (Semi-Implicit Method for Pressure Linked Equations) algorithm is used to solve the fluid control equation. In the DEM model, a circular particle generation factory is set at the entrance, and the stone particle model is simplified into a spherical study (as shown in Fig.3). The particle diameter in the simulation is 10-20 mm and the particle size distribution is random. When the particles flow out of the pipe outlet, the particles are directly removed from the simulation.

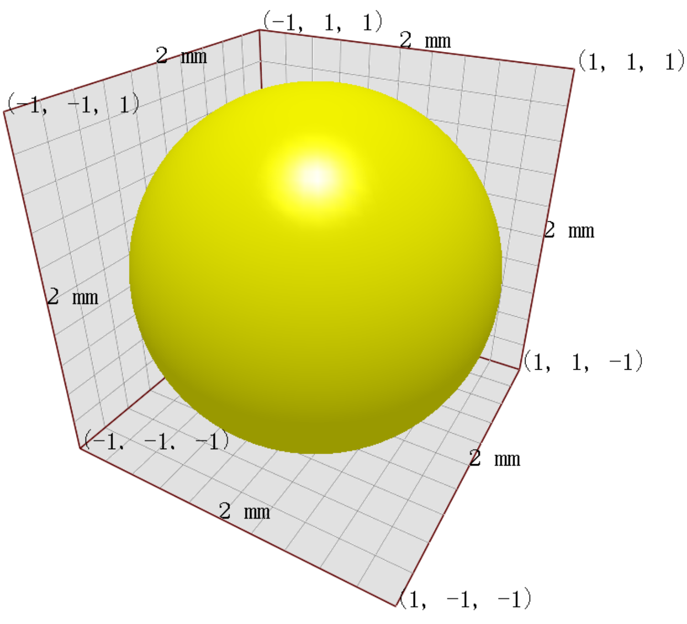

Fig. 3 Spherical particle model diagram

In order to ensure the dynamic accuracy of particle motion information, the time step of CFD should be an integer multiple of DEM, the ratio is 1: 1 ~ 1: 100, and should meet the requirements of Rayleigh time step. Therefore, the time step in DEM is set to 2 × 10-6 s, and the time step in CFD is set to 50 times of DEM, that is, 1 × 10-4 s. The basic conditions of each simulation in this paper are the same, and the physical time of the simulation is set to 10 seconds due to the complexity of the process. Table 1 shows the main parameter settings.

Table 1. Parameter settings in CFD-DEM coupling simulation

parameter	value
fluid density /(kg/m3)	1 150
fluid viscosity /(kg/(m·s))	0.02
fluid velocity /(m/s)	4
particle density /(kg/m3)	2 600
poisson 's ratio of particles	0.3
particle shear modulus /Pa	1.27e+10
particle velocity /(m/s)	2.5
particle flow rate /(kg/s)	19.94
pipe material density /(kg/m3)	7 800
poisson 's ratio of tube wall	0.25
shear modulus of tube wall /Pa	8e+10

4. Simulation results and analysis

4.1. Analysis of slag discharge characteristics of horizontal inclined transportation pipeline

In the slurry shield, the transportation of the slurry pipeline in the horizontal direction is the most common. In this section, by simulating the pipeline transportation process with different inclination angles in the horizontal direction, the dynamic variation law of the slurry and stone particles is studied, which provides a reference for the parameter selection of the slurry circulation system for pipeline laying in the project. The parameters of different simulation cases are shown in Table 2.

Table 2 Parameters of different inclination angles of horizontal transportation pipeline

Simulatio nnumber	1	2	3	4	5	6
Bevel angle α	0°	30°	45°	60°	75°	90°

4.1.1. Mud flow field distribution characteristics

4.1.1.1. Velocity distribution

In order to better observe the transportation characteristics of the pipeline in the horizontal direction, the distribution of the slurry flow field is observed from the perspective perpendicular to the X-Y plane, that is, the transportation law of the slurry is studied from the perspective of the top view. From Figure 4, it can be found that under the horizontal inclined transportation, the velocity of the slurry increases in the near side of the centripetal direction at the elbow of the slurry discharge pipeline, while the velocity decreases in the far side, and it is more obvious with the increase of α value. The overall velocity distribution of pipeline slurry in horizontal inclined transportation is slightly more uniform than that in vertical inclined transportation. This is because the particles are affected by gravity, mostly distributed in the lower part of the pipeline, so the slurry flow in the middle of the pipeline is less hindered.

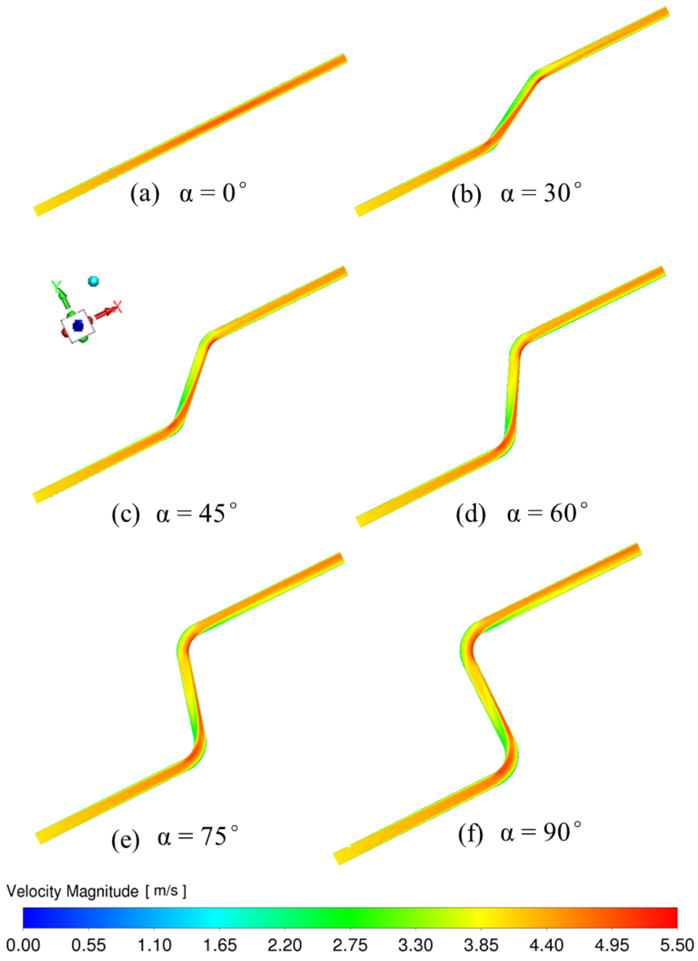

Fig. 4 Mud velocity distribution with different α values

Taking the pipeline transportation with a horizontal inclination angle of 60 ° as an example, the change of slurry velocity distribution before and after adding stone particles was analyzed. According to Figure 5, it can be seen that there is little difference in the distribution of mud flow velocity before and after the particles are added to the pipeline. The main difference is the distribution at the two elbows. The high flow rate area (red part in the figure) of the pipe slurry with particles at the two elbows is stronger than that without particles. This is due to the fact that the stone particles are squeezed to the right side of the pipe when they flow through the first pipe elbow, and the volume flow rate of the fluid in the circular section of the pipe is unchanged, so the mud flow rate on the left side of the pipe is further increased, and the mud flow to the second elbow is the opposite.

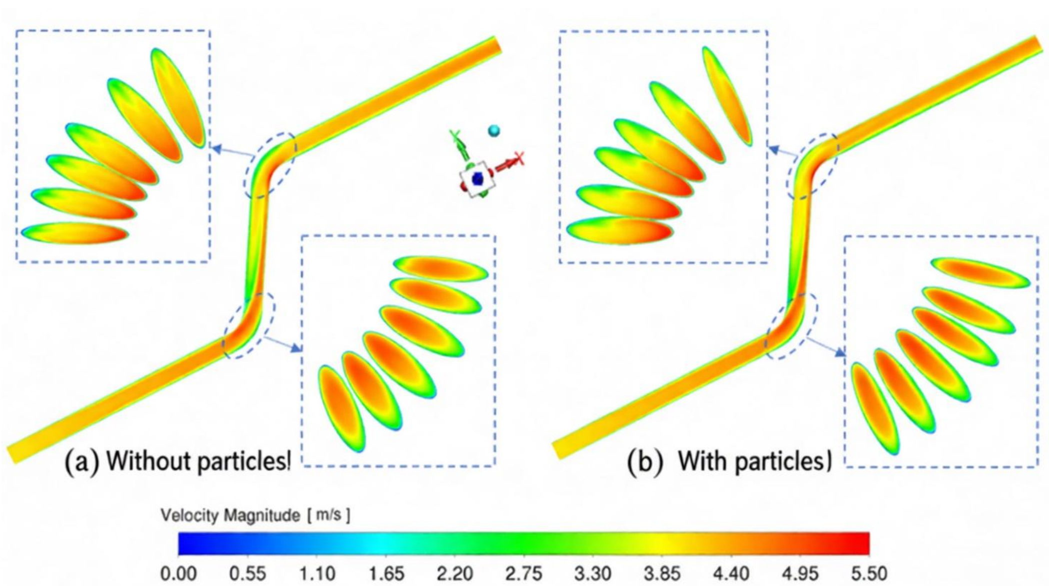

Fig. 5 Comparison of mud velocity distribution details before and after particle addition

4.1.1.2. Distribution of pressure and turbulent kinetic energy

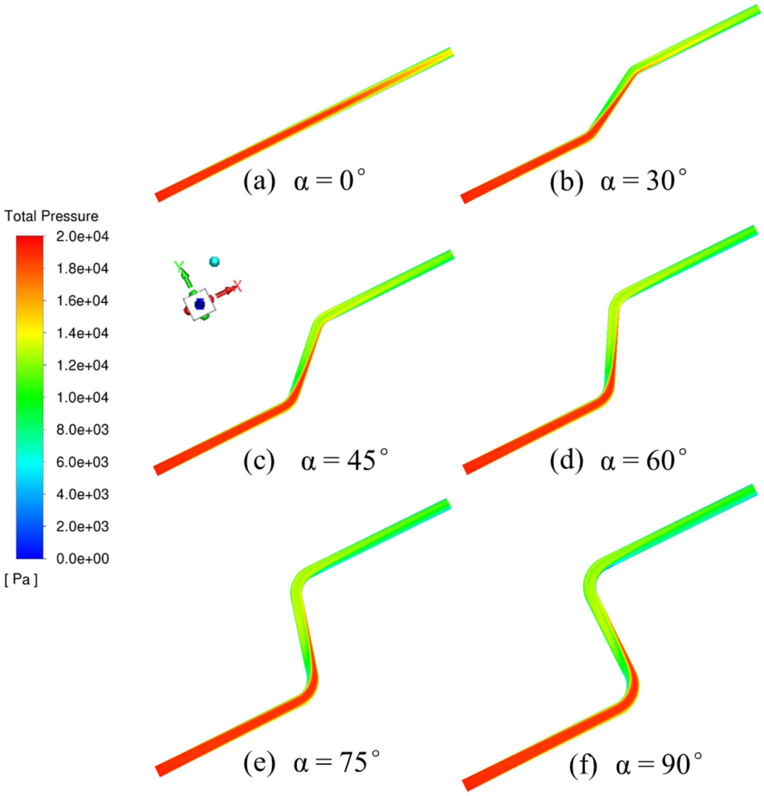

Fig. 6 Mud pressure distribution of different α values

As shown in Figure 6, the total slurry pressure decreases significantly when passing through the first elbow of the slurry discharge pipeline under horizontal inclined pipeline transportation, and it is more obvious with the increase of α value. Because the mud has no gravitational potential energy change, and the loss along the way and the collision loss are less than the vertical transportation.

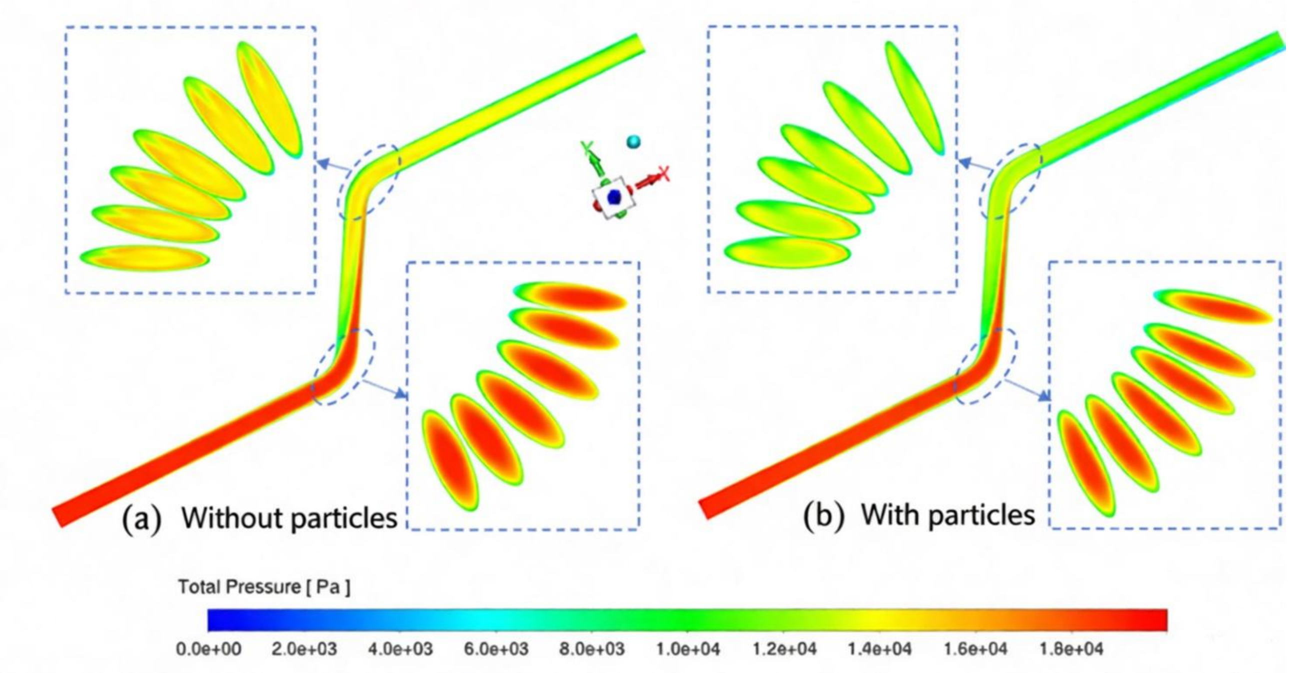

Fig. 7 Comparison of mud pressure distribution details before and after particle addition

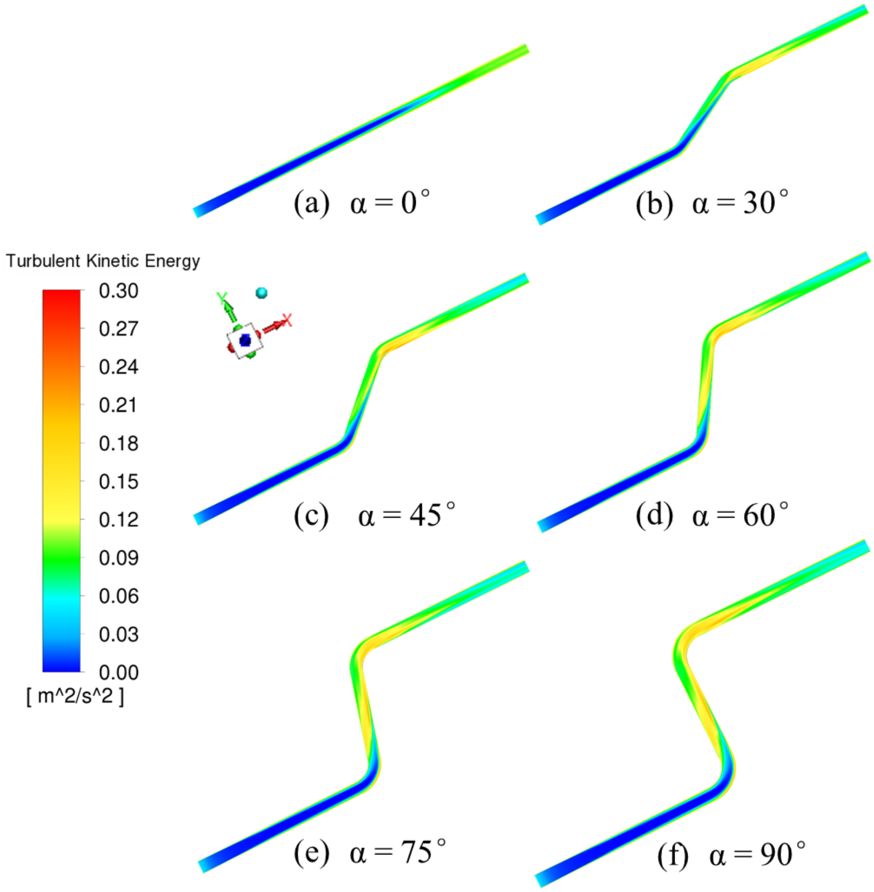

Fig. 8 Distribution of slurry turbulent kinetic energy with different α values

Taking the pipeline transportation with a horizontal inclination angle of 60 ° as an example, the change of slurry pressure distribution before and after adding stone particles was analyzed. As shown in Figure 7, there is no significant difference between the high pressure area of the slurry before the addition of particles (the red part in the figure) and the area after the addition of particles, but when the slurry flows through the middle position of the inclined pipe section, the medium pressure area of the pressure distribution (the yellow-green part in the figure) begins to be quite different. The slurry pressure of the pipeline without particles is greater than that of the particles, and the concentration to the middle of the pipeline is more obvious. From the enlarged diagram of the second elbow of the pipeline, it can be seen that the distribution shape of the slurry pressure without particles is ' V ' and close to the circumferential center of the pipeline, but the slurry pressure with particles is still dispersed in the circumferential outer ring of the pipeline.

It can be seen from Fig.8 that during the transportation of slurry discharge pipelines with different horizontal inclination angles, the distribution of turbulent kinetic energy of slurry mainly differs in the front and rear sections of the second elbow. With the increase of α value, the pulsation degree of fluid micro-groups at this location deepens, the pulsation range is wider, and the turbulent kinetic energy of slurry in the pipeline increases as a whole.

4.1.2. Particle transport characteristics

4.1.2.1. article velocity distribution

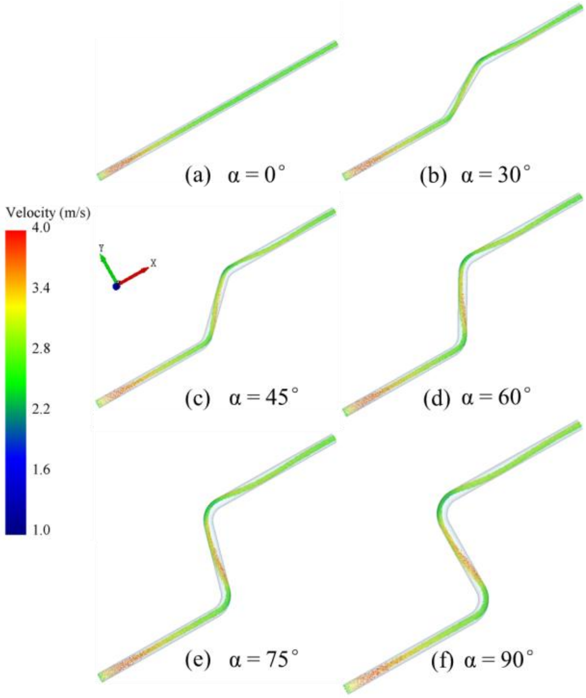

Fig. 9 Particle velocity distribution with different α values

It can be seen from Fig.9 that under horizontal inclined transportation, the speed of stone particles decreases when they are transported to the two elbows of the slurry discharge pipeline. Due to the gravity of the particles, most of the particles are located at the bottom of the pipeline during the transportation process, and will collide with the pipe wall on the side of the elbow, thus changing the horizontal migration direction of the particles at the elbow, so the speed reduction will be less obvious than that in the case of vertical inclined transportation. The accumulation degree of particles in the first elbow is not as serious as that of vertical inclined transportation. Horizontal transportation is more to expand the dispersion degree of particles after collision with the pipe wall at this place, so that some particles are suspended in the horizontal inclined pipe section, which is more conducive to self-acceleration in the mud flow field. Under this influence, with the increase of α value, the number of medium and high speed particles (yellowish red particles in the figure) in the inclined pipe section increases, and the accumulation degree of the second elbow deepens.

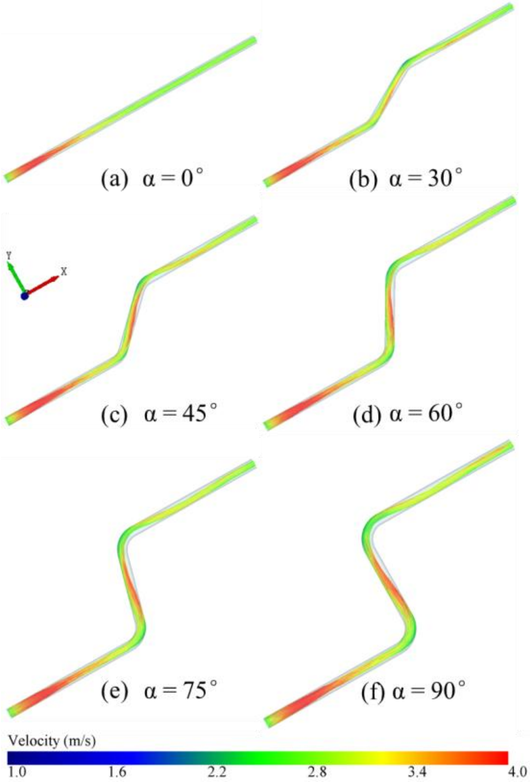

Fig. 10 Particle trajectory distribution with different α values

From Figure 10, it can be seen that there are two obvious acceleration areas in the process of slurry pipeline transportation of stone particles with different horizontal inclination angles. In the first section, when the particles are just loaded into the pipeline, the particles are suspended in the pipeline and are fully driven by the mud, so that the particle trajectory is jet-shaped (red area in the front straight pipe section); the second section is that when the particles flow into the inclined pipe section through the first elbow, as the α value increases, the acceleration effect of the particles is stronger and the acceleration distance is longer. This is because the particle flow collides at the first elbow and disperses from the lower pipe wall position, which is in a flow field position conducive to acceleration. The larger the value of α, the more obvious the dispersion of particles after flowing through the elbow.

4.1.2.2. Degree of particle accumulation at the second elbow

It can be seen from Fig.9 above that the accumulation of stone particles mainly occurs at the second elbow in the slurry discharge pipeline of horizontal inclined transportation. In order to carefully study the blockage of the pipeline, the local details of the particles are compared and observed. It can be seen from Fig.11 that with the increase of α value, the stone particles are closer to the left wall of the elbow, so that the overall position distribution shrinks, which not only increases the friction and wear and collision loss between the particles, but also makes the particles in the pipeline slurry flow field.

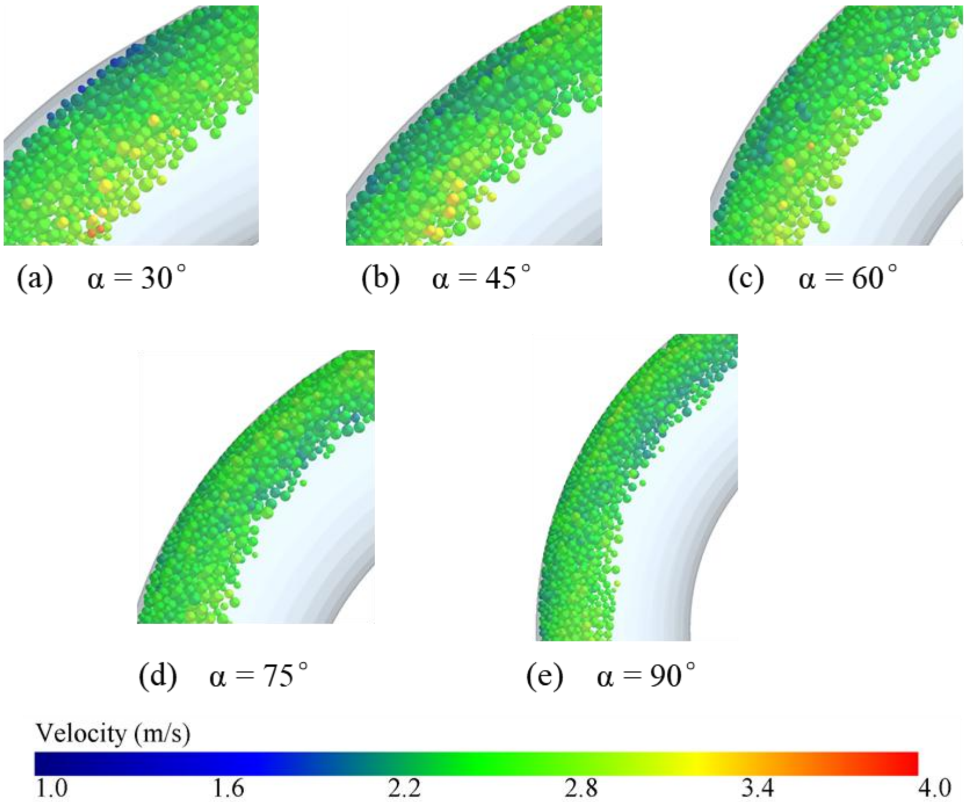

Fig. 11 Comparison of particle accumulation at the second elbow

The particle data at the second elbow are presented separately and analyzed by drawing. As shown in Fig.12, although the average velocity of particles transported by pipeline at different horizontal inclination angles is not much different, the number of particles increases significantly with the increase of α value. Under the value of α from 30 ° to 90 °, the average velocity of particles is 2.57 m / s, 2.49 m / s, 2.51 m / s, 2.53 m / s and 2.48 m / s respectively, and the number of particle accumulation is 406, 659, 915, 1 330 and 1 673 respectively. The average velocity of particles in each horizontal inclined pipeline transportation is not more than 3.62 %, while the number of particles when α is 90 ° is 312.07 % higher than that when α is 30 °.

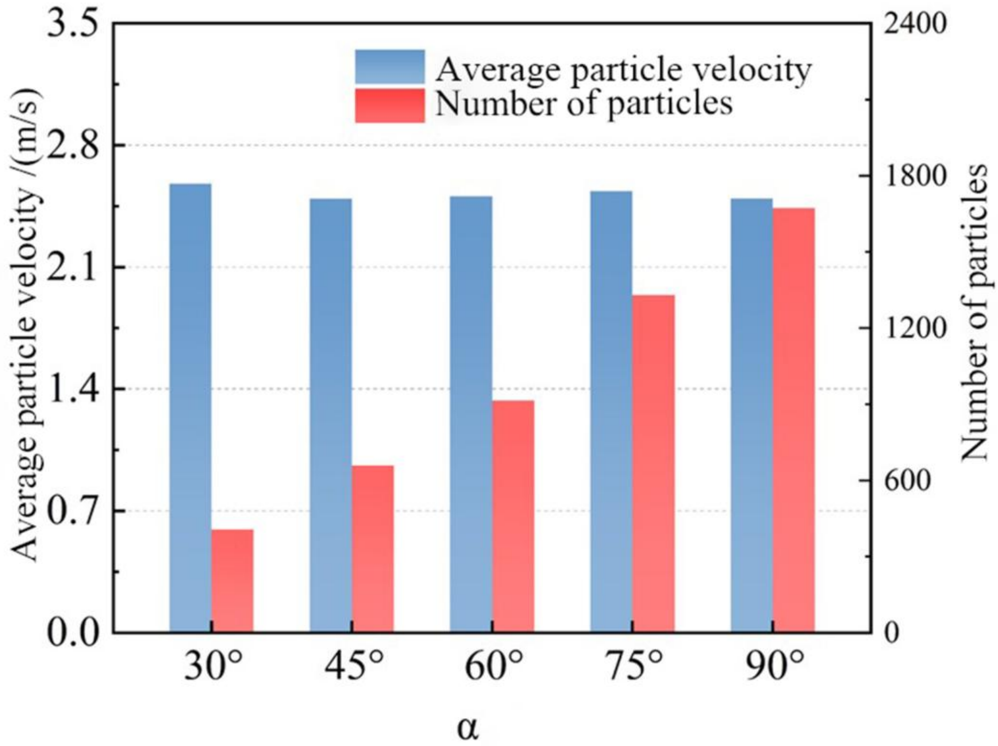

Fig. 12 Comparison of average velocity and number of particles with different α values

Therefore, the larger the horizontal inclination angle of the slurry discharge pipe, the more the accumulation of stone particles at the second elbow. In order to ensure the smoothness of the overall slag transportation of the slurry circulation system, the slurry pipeline laying with different horizontal inclination angles should be reasonably selected according to the actual situation of the project.

5. Summary

5.1. Summary

In this paper, based on CFD-DEM coupling, the horizontal, vertical and inclined slurry discharge pipelines in the slurry circulation system of slurry shield are arranged, and the transportation characteristics of slurry and stone particles are analyzed with different horizontal inclination angles as simulation variables. The conclusions are as follows:

1) In the horizontal inclined pipeline system, when the slurry flows through the elbow area, the velocity distribution of the flow field shows a significant centripetal convergence characteristic: the velocity on the center side of the turning radius is significantly higher than that on the centrifugal side, and the non-uniformity of the velocity distribution gradually increases with the increase of the horizontal inclination angle of the pipeline.

2) From the perspective of energy loss characteristics, the greater the horizontal inclination angle of the pipeline, the more significant the pressure loss of the mud is, which is affected by the coupling of friction and wear along the pipeline, the change of gravitational potential energy gradient and the collision loss between particles. The specific performance is that the pressure difference at the inlet and outlet of the pipeline increases. In terms of turbulence characteristics, the turbulent kinetic energy of the slurry is positively correlated with the horizontal inclination angle of the pipeline; with the increase of the inclination angle, the pulsation degree of the fluid micro-cluster is enhanced synchronously, and the turbulence disturbance characteristics of the flow field are more prominent.

3) In the process of horizontal inclined pipeline transportation, the accumulation effect of stone particles shows significant spatial distribution characteristics, and the particle accumulation phenomenon is mainly concentrated in the second elbow area of the pipeline. The analysis of particle motion parameters under different horizontal inclination angles α shows that the average velocity of particles in the pipeline has little difference under different inclination angles, and the maximum fluctuation range does not exceed 3.62 %. In sharp contrast, the number of particles shows a significant upward trend with the increase of horizontal inclination angle α. The quantitative analysis results show that when α is 90 °, the number of particles in the pipeline is 312.07 % higher than that of α = 30 °. The occurrence of this phenomenon is closely related to the change of gravity component force caused by the change of horizontal inclination angle, the collision frequency between particles and pipe wall and the coupling effect of turbulence characteristics of flow field.

Conflicts of Interest

The authors declare that they have no conflict of interest.

Acknowledgements

Data Availability Enquiries about data availability should be directed to the authors.

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