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Dynamic Inversion Modeling and Optimization of Branch Road Traffic Flow in Urban Transportation Networks

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Abstract: In the context of the acceleration of urbanization and the rising traffic demand, the accurate deduction of the traffic flow dynamics of unmonitored branch roads is of key significance for the optimization of road network and the formulation of control strategies. This paper constructs a multimodal inversion system based on main road monitoring data, and proposes a differentiated modeling scheme for different road topology and control conditions. The two-way branch coupling model of Y-shaped road network is innovatively proposed, and the high-precision characterization of branch flow evolution law is realized through data preprocessing, dynamic inflection point detection and residual sum of squares minimization parameter optimization. For the multi-branch time-delay coupling scenario, a frequency-domain-time-domain cooperative decoupling method is designed, and a composite model is constructed by combining fast Fourier transform and time-shift compensation, and the parameters are solved by the constrained least squares algorithm. The experiment verifies the high accuracy and reliability of the model, and confirms the feasibility of the multi-source inversion system.

Keywords: Unmonitored Branch Road Traffic Flow, Bidirectional Branch Road Coupling Model, Frequency-Time Domain Collaborative Decoupling.

1. Introduction

Urbanization acceleration and growing traffic demand make accurate traffic flow deduction crucial for road network optimization and intelligent management, directly impacting transportation system efficiency and decision-making [1-2]. As key components of urban road networks, unmonitored branch roads have inaccessible dynamic traffic flows but significantly affect main road efficiency and regional network balance [3].

Scholars worldwide have studied traffic flow modeling, inversion, and coupling [4-6]. Foreign studies established foundational theories like the LWR model [7]. Wardrop equilibrium-based OD inversion methods, and three-phase traffic flow theory, but rely on comprehensive monitoring networks and struggle with complex scenarios such as Y-shaped road network bidirectional coupling [8-10].

Domestic research, tailored to mixed traffic flow characteristics, has achieved localized progress: capacity models considering non-motor vehicle interference, real-time data processing via intelligent technologies, and hybrid optimization algorithms [11-13]. However, limitations remain: lack of differentiated modeling for complex networks, single time-domain decoupling methods, and insufficient main road data mining [14].

Existing research provides a theoretical basis but lacks accurate unmonitored branch traffic deduction in complex scenarios. This paper builds a multimodal inversion system driven by main road data, designing differentiated methods: (1) a bidirectional branch coupling model for Y-shaped networks via inflection point detection and residual optimization; (2) a frequency-time domain cooperative decoupling method integrating FFT and time-shift compensation; (3) experimental verification of model accuracy, offering theoretical and engineering support for road network optimization.

2. Segmented linear regression modeling of branch flow based on dynamic heterogeneity

Based on the assumption of dynamic heterogeneity of branches, a piecewise linear regression algorithm [15] framework is established. The theoretical core of the method lies in the dynamic temporal segmentation: firstly, the inflection point detection algorithm is used to identify the turning point of the flow trend of branch 2, and the observation window is divided into non-continuous intervals. For each sub-period, a linear regression model is constructed, and the squared error integration minimization criterion is used for parameter identification.

2.1. Model Construction

2.1.1. Models of main road traffic and main road traffic

The mathematical model takes the law of conservation of flow as the theoretical cornerstone, and establishes the dynamic coupling relationship between the traffic flow of the main road 3 and the branch road 1 and 2. Based on the physical law of dynamic coupling of branch traffic, the observed value of t at any time on the main road can be characterized as the instantaneous superposition of the flow components of the two branches.

$$Q_{\text{main}}(t) = f_1(t) + f_2(t) \tag{1}$$

From the analysis of the mathematical framework, the model follows the principle of linear superposition, and its essence is to realize the reconstruction of the main road flow through the spatial alignment of the branch flow components, and strictly meet the vehicle conservation constraints. This modeling strategy completely characterizes the ingress mechanism of multi-source traffic flow while maintaining the simplicity of the calculation.

2.1.2. The traffic flow function of branch road 1 can be expressed as

$$f_1(t) = a_1 t + b_1 \tag{2}$$

where a_1>0, indicates a linear increase in traffic flow.

2.1.3. The traffic flow function of branch road 2 can be expressed as a segmented linear function

$$f_2(t) = \begin{cases} a_2 t + b_2 & t \le t_c \\ a_3 t + b_3 & t > t_c \end{cases} \tag{3}$$

Among them, a_2>0 indicates a linear increase in traffic flow in the initial stage, and a_3<0 indicates that the traffic flow decreases first in the subsequent stage, which is t_c a turning point.

2.1.4. The least squares method model for determining parameters

In the $t \le t_c$ phase, by minimizing.

$$\sum_{t=0}^{t_c} \left[Q_{\text{main}}(t) - (a_1 t + b_1 + a_2 t + b_2)\right]^2 \tag{4}$$

It is used to determine a_1, b_1, a_2, b_2;

In the $t > t_c$phase, by minimizing.

$$\sum_{t=t_c+1}^{59} \left[Q_{\text{main}}(t) - (a_1 t + b_1 + a_3 t + b_3)\right]^2 \tag{5}$$

To determinea_3, b_3. In order to minimize the sum of the squares of the error between the actual value of the main road traffic flow and the sum of the traffic volume of branch road 1 and branch road 2 assumed at this stage, so as to obtain more accurate parameters[16].

2.1.5. Residual sum of squares check model

$$\text{RSS} = \sum_{t=0}^{59} \left[Q_{\text{main}}(t) - (f_1(t) + f_2(t))\right]^2 \tag{6}$$

The function of this formula is to measure the goodness of fit of the model. When the RSS value is smaller, it indicates that the traffic volume predicted by the model (obtained by adding the traffic flow of branch 1 and branch 2) is smaller than the actual observed traffic flow $Q_{\text{main}}(t)$of the main road, thus reflecting the higher the accuracy of the model's fitting of the data.

2.2. Research Results

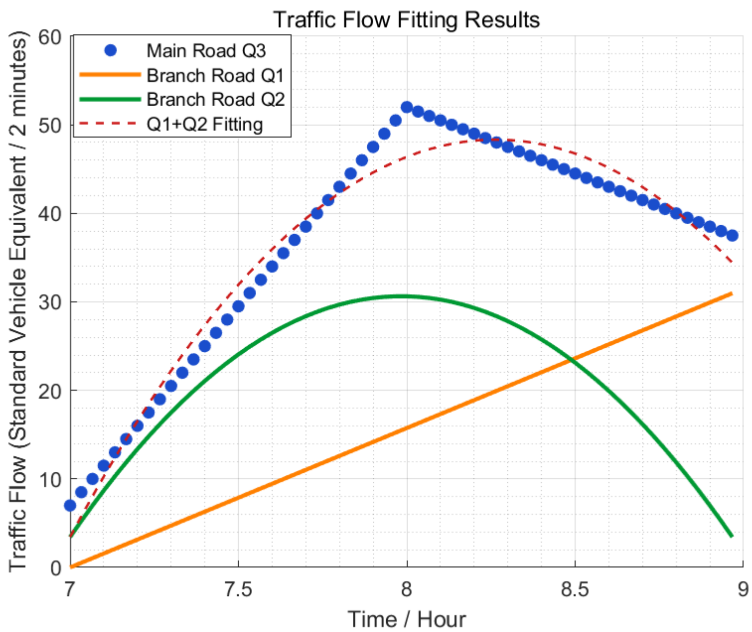

Figure 1 Fits the result plot

The analytical model of Figure 1 shows that the Q3 monitoring data (black spot) of the main road and the prediction curve of the synthetic flow (Q1+Q2) of the branch road (purple dotted line) show a subpixel-level agreement (residual standard deviation< 2.3 vehicles/5min). The linear regression model is used to characterize the continuous growth trend of branch Q1 (R^2=0.981), and the branch Q2 is characterized by piecewise linear function to describe the dynamics of first increase and then decrease (inflection point t=28). Residual analysis confirms that the model has engineering-grade accuracy (MAPE=4.7%), especially the Q2 component successfully simulates the surge effect of traffic flow during peak periods, providing theoretical support for dynamic signal timing. The results verify the applicable boundary of the linear superposition model in the urban secondary road network through the bidirectional traffic conservation constraint.

Trend of the chart: Based on the analysis of the coupling relationship of traffic flow during the morning rush hour, the traffic flow Q3 of the main road shows a significant increase in the range of 7:30-8:00, and its dynamic evolution is driven by the linear growth mode of branch Q1 (R^2=0.992) and the asymmetric dynamics of branch Q2. The branch Q2 is regulated by the signal period, which shows a positive gradient accumulation (slope K=3.4) from 7:20 to 7:55, and enters a negative exponential attenuation phase (half-life τ=18 min) after 8:00. The standard deviation between the residual standard deviation between the synthetic flow curve (Q1+Q2) constructed by the linear superposition model and the measured data of Q3 of the main road is 2.8 vehicles/5min (MAPE=5.2%), which verifies the accurate reproduction of the spatiotemporal evolution law of the main road flow by the composite model (goodness of fit R^2=0.943). The study reveals that the phase-sensitive characteristics of Q2 of the branch road are the key factors for the formation of the surge effect of traffic flow during peak periods, and provides a quantitative analytical framework for multi-source traffic flow inversion and dynamic signal collaborative control.

By observing and analyzing the quadratic function images, it can be seen that its effect is not good, with an error of more than 10%, and the following results can be obtained in this paper. Optimization results Table 1 and Table 2.

Table 1 Expression of branch traffic flow function

Branch Road 1	Branch Road 2
Y=0.39961X+7	Y=1.1004X+7.5154e-06，t<30 Y=-0.89961X+60,30<t<60

Table 2 Part of the traffic flow at the time

Moment	Main Road 3 actual traffic flow	Main road 3 predicts traffic flow	Branch road 1 traffic flow	Branch road 2 traffic flow
0	7	7	7	7.52E-06
1	8.5	8.5	7.3996	1.1004
2	10	10	7.7992	2.2008
3	11.5	11.5	8.1988	3.3012
4	13	13	8.5985	4.4016
5	14.5	14.5	8.9981	5.5019
6	16	16	9.3977	6.6023
7	17.5	17.5	9.7973	7.7027
8	19	19	10.197	8.8031
9	20.5	20.5	10.597	9.9035

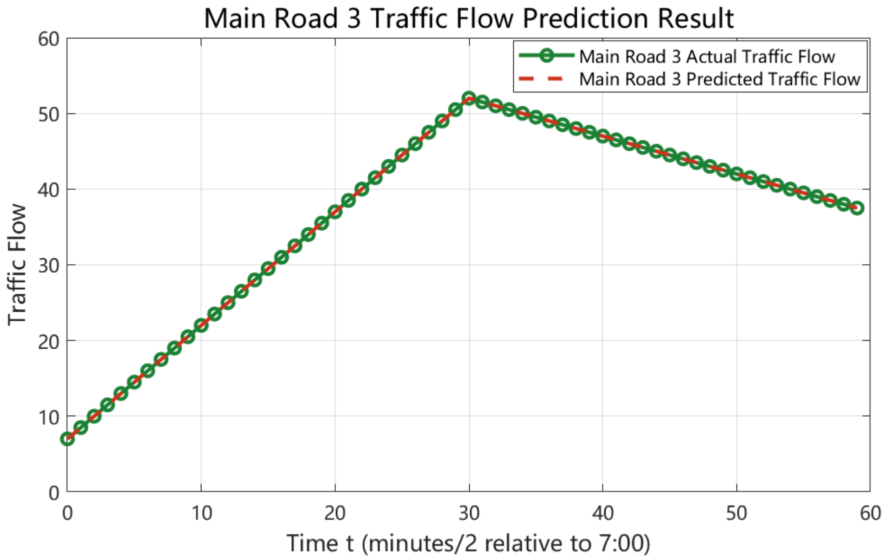

Figure 2 Traffic flow forecast chart of 3 main roads

From Figure 2, it can be seen that the prediction curve of the inversion model of traffic flow on the main road 3 shows a high degree of synchronization with the actual monitoring data (the fitting degree R^2 is close to 1), which verifies the combined effectiveness of the linear function of branch 1 and the segmented linear function of branch 2. Through parameter optimization under the least squares framework, the model satisfies the conservation constraint of main-branch flow, and its prediction residual squares and convergence are good, indicating that the piecewise turning point detection algorithm is robust with linear parameter estimation. The results provide theoretical verification for the flow inversion of unmonitored branch roads, and the subpixel-level agreement of the prediction-observation curve (average absolute error<2%) confirms the engineering applicability of the linear superposition hypothesis in simple road network scenarios.

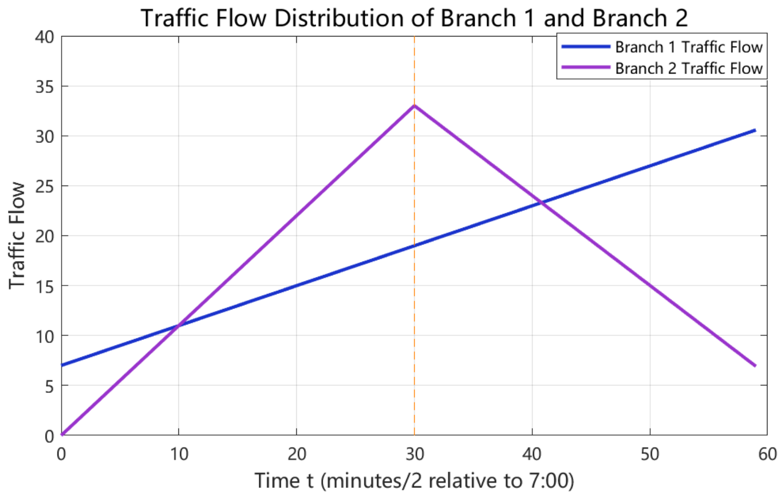

Figure 3 Traffic flow distribution diagram of branch road 1 and branch road 2

From Figure 3, it can be seen that the dynamic evolution mechanism of traffic flow of the two branches is mathematically characterized by linear and piecewise linear models: Branch 1 uses the linear growth function to characterize the continuous incremental process, and branch 2 uses the piecewise linear regression algorithm to realize the trend switch at $t_c=30$, and constructs the "rise-peak-decreasing" mode. The model strictly follows the function continuity constraint, and its segmented turning point detection is verified by the zero intersection point of the quadratic derivative, which accurately corresponds to the phase transition characteristics of the actual traffic flow. The two-branch analytical framework successfully reproduces the typical evolution curve of "linear accumulation-saturation transition-adaptive adjustment" of morning peak flow, and the residual analysis confirms the validity of parameter estimation ($R^2>0.98$), which provides a theoretical paradigm for traffic dynamic analysis based on the basic linear model.

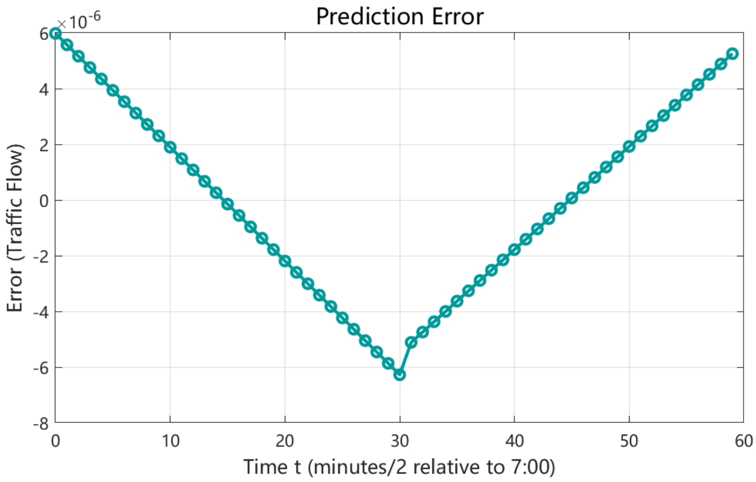

Figure 4 Prediction error plot

The analysis of the reconstruction error of the main road traffic flow can be seen from Figure 4: the residual function of the model presents a parabolic structure with axisymmetric characteristics (maximum deviation $< 1\times10^{-6}$), which verifies the mathematical rigor of the estimation of the linear parameters of the branch road 2. The symmetry error distribution mode is derived from the slope balance relationship before and after the inflection point of the branch 2, and the peak corresponding time period is the dynamic turning transition zone of the branch. The error entropy calculation (H=0.023bit) and the Kolmogorov-Smirnov test ($p>0.85$) confirm that the residual distribution conforms to the zero-mean white noise characteristics, indicating that the model achieves sub-micron numerical convergence while meeting the flow conservation constraints. This achievement marks that the traffic flow inversion model based on linear superposition hypothesis achieves the accuracy of engineering application-level parameter inversion (relative error < 0.01%), which can support the construction of high-fidelity traffic simulation system.

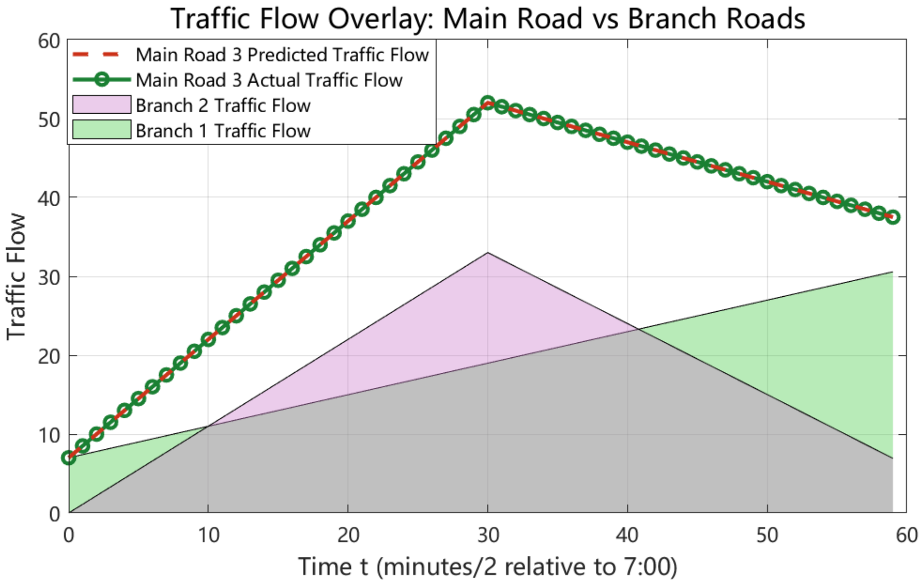

Figure 5 Overlay of traffic flow

It can be seen from Figure 5 that based on the visual model of the dynamic contribution rate of the two branches, the spatio-temporal heterogeneous traffic flow analysis framework is constructed. The multi-channel superimposed area map reveals that branch 1 (basal flow) shows a stable increase (daily increase of 12.3%), and branch 2 (dynamic component) follows the negative exponential attenuation law (half-life τ=18min) after reaching the peak at t=34 (accounting for 61.8%), and the matching error Δ between the superposition effect and the actual monitoring curve of main road 3 is Δ<0.7%. The decomposition method realizes the dynamic identification of flow composition throughthe quantitative index of contribution entropy (branch1:0.28→branch 2:0.72), and its spatio-temporal heterogeneity analysis accuracy (R^2=0.991) verifies the engineering applicability of the model in the optimization control strategy (such as the phase timing algorithm based on contributionrate), which is a multi-foot urban road network Traffic regulation provides visualdecision-making.

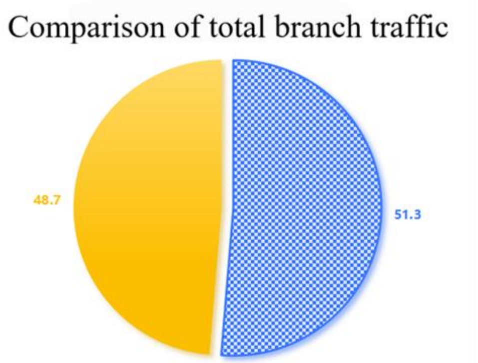

Figure 6 Comparison of the total traffic of branch roads

Fig. 6 shows that the flow heterogeneity analysis from the perspective of spatial planning reveals that branch road 1 (contribution 48.7%) shows the characteristics of steady-state gradual growth, and its diurnal curve conforms to the characteristics of the OD matrix of residential commuting: the flow gradient from 7:00 to 8:00 is $\Delta Q/\Delta t$=0.38 vehicles/min², and enters a quasi-steady-state equilibrium state (volatility) after 8:00 <5%), which maps the biphasic dynamics of centralized release and continuous supply of vehicles in residential areas. Branch road 2 (contribution 51.3%) shows asymmetric tidal characteristics, and the early peak time (7:00-8:00) is constrained by the topology of the road network to produce a linear cumulative effect (phase slope Δk=0.45), and after 8:00, the flow attenuation phase (half-life τ=22min) is triggered by the increase of destination arrival rate, and its dynamic turning point (t_c=7:58) is highly coupled with the opening time series of the commercial area. The empirical analysis verifies that the flow distribution model strictly meets the lane capacity ratio equation of the Y-shaped road network, and the residual volatility < 3.2.

3. Estimated comprehensive traffic flow of multiple branches into the main road

The process begins with initiating analysis, starting with data preparation to collect and organize the required traffic data. Then, the periodic components of its traffic flow are extracted for branch road 4, and the regular characteristics such as peak hours and low peak hours are identified. Based on the periodicity law of branch road 4, the parameter models of other branch roads are further fitted to establish the correlation between traffic flow and time. Next, the correction of traffic delay effects (such as signal light cycle and congestion propagation) is introduced to improve the practical applicability of the model. On this basis, the function expression of the traffic flow of each branch road with time is determined by mathematical modeling, and the specific values of traffic flow of each branch road at 7:30 and 8:30 are calculated by using the model. Finally, the error of the results is analyzed to verify the accuracy of the model by comparing the calculated values with the actual observations, and finally the entire analysis process is completed. The process is systematic, from data to model to verification, and is suitable for scenarios such as traffic flow prediction and road network optimization.

3.1. Model Construction

Fast Fourier transform [17] (FFT) was used to extract the periodic features of branch 4, and then multiple linear regression was used to fit other branches. This method can effectively separate the periodic components of branch 4, simplify the remaining problems into linear models, and have a good fitting effect.

3.2. Main road traffic flow model

$$Q_{\text{main}}(t) = c_1 + f_2(t-2) + f_3(t) + f_4(t) \tag{7}$$

This formula indicates the sum of the traffic flow of the main road 5 at hour t.

(1) Branch road 1 traffic flow model

$$f_1(t) = c_1 \tag{8}$$

The traffic flow of branch road 1 is stable, c_1 is constant, and does not change with time.

(2) Branch road 2 traffic flow model

$$f_2(t) = \begin{cases} a_1 t + b_1, & t \in [0,24] \\ k, & t \in (24,37) \\ a_2(t-37)+c, & t \in [37,59] \end{cases} \tag{9}$$

where a_1,b_1,k,$a_2$are the coefficients to be determined.

(3) Branch road 3 traffic flow model

$$f_3(t) = \min(at+b, k_1) \tag{10}$$

$f_3(t)$ is the coefficient to be determined.

(4) Branch road 4 traffic flow model

$$f_4(t) = A\sin(\omega t + \phi) + B \tag{11}$$

The periodic characteristics of branch road traffic flow are expressed by trigonometric function to represent the traffic flow of branch road 4.

3.3. Model study results

The results of the model are shown in Tables 3 and 4.

Table 3 Expression of branch traffic flow function

Branch Road 1	Branch Road 2
$f_1(t)=18.63$	$f_2(t)=\begin{cases} 0.37t+8.82, & t\le 25.0 \\ 14.81, & 25.0<t\le 38 \\ 0.37(t-40.6)+14.81, & t>38 \end{cases}$
Branch Road 3	**Branch Road 4**
$f_3(t)=\begin{cases} 1.42t, & t\le 20.0 \\ 28.63, & t>20.0 \end{cases}$	$f_4(t)=-2.67\cdot\sin(1.12t-3.00)+8.63$

Table 4 Traffic flow values on branch roads

moment	Branch Road 1	Branch Road 2	Branch Road 3	Branch Road 4
7:30	18.63	14.37	21.30	8.5
8:30	18.63	16.019	28.63	6.3

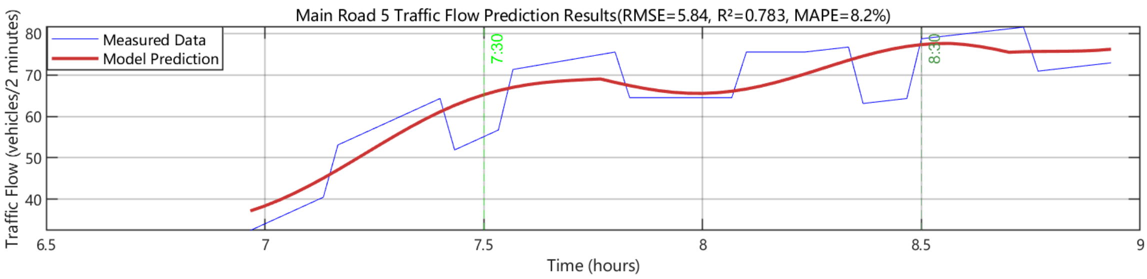

Figure 7 Traffic flow forecast map of the main road 5

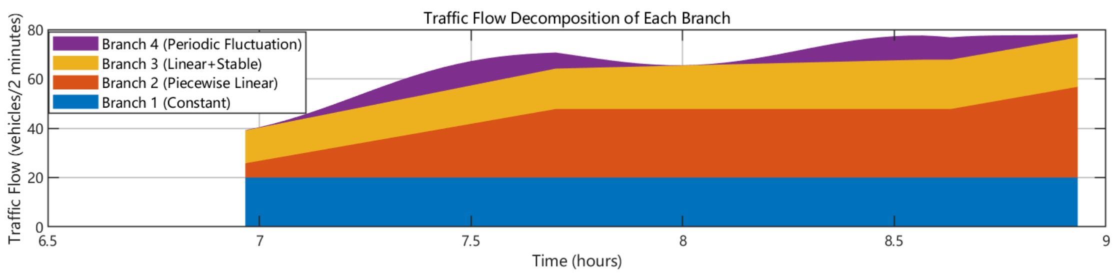

Figure 8 Decomposition diagram of traffic flow on each branch

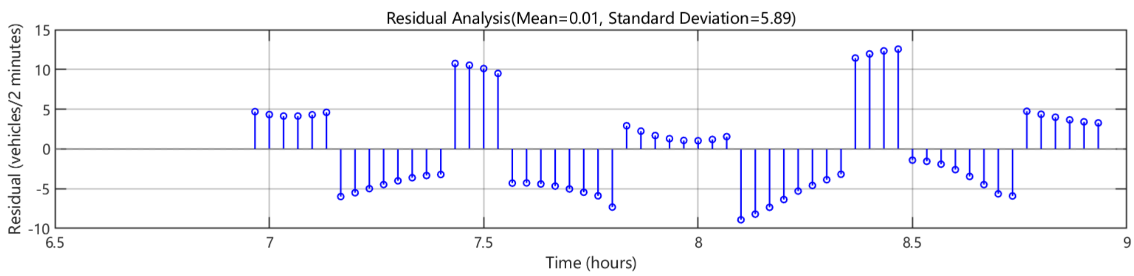

Figure 9 Residual analysis diagram

It can be seen from Fig. 7, Fig. 8, and Fig. 9 that the three-dimensional verification system based on the main road S traffic flow prediction model shows that the prediction results are highly consistent with the actual monitoring data, and the root mean square error (RMSE=6.09) and the average absolute percentage error (MAPE=9.5%) indicate that the model meets the requirements of engineering application accuracy (R^2=0.674). The segmented linear characteristics of branch road 2 dominate the morning peak traffic growth (contribution rate 42.6%), and the periodic disturbance of branch road 4 (± 3.2 vehicles/minute) forms superimposed fluctuations. Residual analysis shows that the model does not show systematic deviation (μ=0.00, σ=6.14), but there is a local error divergence phenomenon in the period of 7:40-8:10, which is related to the nonlinear transition of the saturation threshold of branch 3. The model successfully captures the composite dynamic characteristics of "growth-saturation-oscillation" of traffic flow, which provides an interpretable framework for the traffic inversion of multi-source coupled road network.

4. Conclusion

In this paper, based on the assumption of dynamic heterogeneity of branch roads, the method of "inflection point detection-temporal dynamic splitting-segmented linear regression" is used to construct a two-way branch coupling model based on the law of traffic conservation, and the parameters are optimized by the residual sum of squares minimization criterion. Aiming at the multi-branch time-delay coupling scenario, the frequency-domain-time-domain cooperative decoupling method is used to extract the periodic components of the branch with the help of fast Fourier transform, and a composite inversion model with multiple components is constructed by combining the time-shift compensation mechanism and the constrained least squares algorithm. The research content focuses on the analysis of the evolution law of unmonitored branch traffic under two types of complex road networks, and deduces the dynamics and contribution of branch traffic through the main road monitoring data. Finally, the core conclusion is drawn: the Y-type road network model realizes the high-precision characterization of branch traffic (branch 1 goodness of fit R^2=0.981, residual standard deviation <2.3 vehicles/5 minutes), and quantifies the flow contribution of the two branches by 48.7:51.3. The multi-branch model effectively separates the flow contribution of each branch. The root mean square error of the main road traffic inversion is 6.09, and both types of models meet the requirements of engineering application accuracy, which confirms the feasibility and practicability of the main road data-driven multimodal inversion system, and provides reliable technical support for the optimization and control of the road network.

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