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Synergistic Optimization of Competitive Performance and Commercial Profitability in Professional Women's Basketball

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Abstract: Addressing the operational challenge faced by professional women's basketball clubs in balancing competitive performance and commercial profitability under salary cap constraints, this study constructs an integrated multi-model dynamic decision-making system. Using the Connecticut Sun as a case study, the research first employs the Entropy Weighted Topix method to establish a player evaluation system encompassing competitive contribution, commercial value, and dynamic risk factors, enabling precise stratification of player value. Subsequently, a BP neural network is utilized for nonlinear forecasting of ticket, sponsorship, and merchandise revenues, combined with a Markov model to simulate the cross-seasonal evolution logic of team status. The core process employs multi-objective optimization using the NSGA-II algorithm to identify Pareto-optimal operational solutions, ensuring synergistic enhancement of competitive strength and economic returns while maintaining a 90.01% salary cap utilization rate. Addressing external uncertainties, the study quantifies potential financial risks from key player injuries and macroeconomic fluctuations via Monte Carlo simulations, while graph theory analyzes the win rate enhancement effects of team collaboration networks. Empirical results indicate that retaining the current 12 players yields an annual net profit of approximately $2.29 million and a projected 60% win rate under the baseline scenario. Through data-driven modeling, this study provides robust quantitative recommendations for resource allocation in professional sports.

Keywords: Professional sports operations; Multi-objective optimization; Risk assessment model.

1. Introduction

In the commercial landscape of modern professional sports, maximizing both competitive achievements and financial returns within limited resource allocations has become a core challenge for management. With the explosive growth of the women's basketball league's influence, clubs now face multiple challenges: strict salary cap constraints, heightened player rights awareness, and intensified market competition. Managers must not only identify athletes with dominant on-court performance but also assess their ability to monetize engagement across social media and community participation. Previous academic research has predominantly focused on single-dimensional athletic metric evaluations or static financial statement analysis, often overlooking the dynamic disruptions to system stability caused by external variables such as injury risks and league expansion[1-2]. The innovation of this section lies in proposing a closed-loop intelligent decision-making logic that transforms discrete player performance data into continuous profit-loss prediction curves, incorporating a risk control mechanism based on probability distributions. The overall research framework encompasses six mutually reinforcing subtasks: First, constructing a multidimensional player value assessment system to quantify both technical and market contributions; second, establishing a revenue prediction model; followed by state evolution simulations[3-4]; then developing multi-objective optimization strategies under salary constraints; conducting risk assessments under extreme external shocks; and finally analyzing team collaboration effectiveness through graph theory. Through this multi-stage modeling process, this research aims to provide a practical algorithmic reference for digital governance in professional sports[5].

2. Model Construction

2.1. Overall Modeling Framework

Brief Description: To address the four sub-problems, we have constructed an integrated decision system consisting of six mutually supporting sub-models. A system block diagram can be used to illustrate the input-output relationships between the six models (e.g., Player Value → Revenue Prediction → State Prediction → Strategy Optimization).

2.2. Indicator System and Constraint Condition Setting

2.2.1. Data Sources

The data used in this study primarily comes from publicly available professional sports statistical databases, financial analysis platforms, and governmental macroeconomic statistical agencies. The dataset includes multidimensional player performance metrics, salary structures, team commercial revenues, and city economic background data, providing multisource heterogeneous support for model construction[6-7]. Specific data sources are shown in table 1:

Table 1 Data Sources

Source	Description
basketball-reference.com	Provides player performance indicators (e.g., PER, WS, usage rate) and historical injury records.
spotrac.com	Contains detailed player contract amounts, team salary cap allocations, and league salary cap rules.
statista.com	Provides ticket revenue, media contract valuations, and sponsor data for the WNBA and related leagues.
census.gov	Supplies macroeconomic indicators for city-site analysis, including population size, per capita disposable income, etc.

2.2.2. Data Preprocessing

To ensure the accuracy of the dynamic decision model, we performed cleaning and standardization on the raw data. First, for missing values in player performance data, if the missing proportion is below 5% we use mean imputation; otherwise, the sample is discarded. Second, we apply the 3σ principle to identify and remove outliers in the financial data. Because the scales of athletic performance (e.g., points) and commercial value (e.g., social media following) differ, to eliminate scale effects, we adopt Min-Max normalization to map each indicator onto the [0,1] interval[8-9]. Let the original indicator be x_i, the resulting dimensionless indicator z_i is computed as:

$$z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)} \tag{1}$$

Furthermore, for injury risk analysis we apply a time-weighted treatment to historical missed game counts, ensuring that recent data receive higher weight in risk assessment. After processing, the dataset is transformed into a standardized matrix suitable for the entropy weight method and the dynamic optimization model encoding the prior of subsequent evaluation logic[10].

2.2.3. Competitive Performance Indicators

Table 2 Classification and Explanation of Competitive Performance Indicators

Indicator Type	Specific Indicator Examples	Calculation Method and Significance
Basic Statistics	Points, Rebounds, Assists, Steals, Blocks	Direct statistics reflecting basic on-court contributions
Advanced Efficiency Metrics	PER (Player Efficiency Rating), WS (Win Shares), True Shooting Percentage	Standardized calculations quantifying marginal contribution to team wins
Situational Performance Indicators	Clutch Scoring, Pressure Defense Efficiency	Statistics in specific time/scenarios capturing performance differences in critical moments
Team Compatibility Indicators	Tactical Fit, Usage Rate-Efficiency Balance	Calculated with team's tactical system to evaluate integration capability

Classification and Explanation of Competitive Performance Indicators are shown in table 2. This dimension includes not only basic statistical data such as points and rebounds but also introduces advanced metrics like PER (Player Efficiency Rating) and WS (Win Shares) to measure players' marginal contribution to victories. Additionally, we consider players' performance in critical moments (e.g., fourth-quarter scoring efficiency) and their compatibility with the team's existing tactical system.

2.2.4. Commercial Value Indicators

This dimension goes beyond traditional jersey sales, integrating social media engagement (Instagram and TikTok followers and interactions), local media exposure, and the pull effect on premium ticket sales. We use the entropy weight method to weight these sub-indicators and ultimately synthesize a unified commercial rating F_i.

2.2.5. Economic Environment and Risk Indicators

Table 3 Economic Environment and Risk Indicators

Indicator Type	Specific Indicator Examples	Calculation Method and Significance
Macroeconomic Indicators	GDP growth rate, household entertainment consumption expenditure, inflation rate	Public macroeconomic data, reflecting the supporting capacity of the external environment.
Industry Environment Indicators	Broadcasting contract value, salary cap adjustment range, sponsor renewal rate	Public data from the league, reflecting industry development trends.
Risk Indicators	Player Injury Probability, Game Fluctuation Risk, Fan Consumption Willingness Volatility	Fitting probability distribution based on historical data to quantify potential impact likelihood and severity.

Economic Environment and Risk Indicators are shown in table 3. This dimension places the team within a broader macro environment. We introduce GDP growth rate as a proxy variable for residents' willingness to spend on entertainment and fit players' personal injury history data into a probability distribution to quantify their future injury risk.

2.2.6. Model Constraint Settings

All models must operate under strict constraint conditions.

(1) League Rule Constraints:

Salary Cap: $\sum_{i=1}^{n} s_i x_i \le S_{cap}$ (where $S_{cap} = 1{,}300{,}000$ USD)

Roster Size: $10 \le \sum_{i=1}^{n} x_i \le 15$

Position Configuration: $\sum_{i\in G} x_i \geq 3$, $\sum_{i\in F} x_i \geq 3$, $\sum_{i\in C} x_i \geq 2$ (G=Guards, F=Forwards, C=Centers)

(2) Economic and Operational Constraints:

Cash Flow Health: Monthly operating expenses shall not exceed 80% of monthly revenue.

Return on Investment: The expected annualized return rate for any major investment (e.g., venue upgrades) must exceed 8%.

(3) Risk Control Constraints:

Insurance Coverage: All key players must purchase injury insurance covering their annual salary.

Revenue Diversification: Revenue from any single source (e.g., television broadcasting) shall not exceed 60% of total revenue.

2.3. Key Model Construction

To address the four sub-problems, we have deployed six mutually supporting core models, forming a complete decision-making loop from value assessment to strategy optimization. The specific construction process of each model is as follows:

2.3.1. Comprehensive Player Value Evaluation: Entropy Weight Method + TOPSIS

Step 1: Data Standardization

The Z-score method is used to standardize all original indicators (competitive performance S_i, commercial value F_i, injury risk R_i) to eliminate dimensional effects:

$$z_{ij} = \frac{x_{ij} - \mu_j}{\sigma_j} \tag{2}$$

Where μ_j is the mean of the j-th indicator, σ_j is the standard deviation of the j-th indicator.

Step 2: Entropy Weight Calculation

Calculate the entropy value of the j-th indicator:

$$e_j = -\frac{1}{\ln n}\sum_{i=1}^{n} p_{ij}\ln p_{ij} \tag{3}$$

Where $p_{ij} = \frac{z_{ij}+1}{\sum_{i=1}^{n}(z_{ij}+1)}$ (to avoid ln0), n is the number of players. Then obtain the objective weight:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{m}(1 - e_j)} \tag{4}$$

Where m is the number of indicators, effectively avoiding subjective weighting bias.

Step 3: TOPSIS Ranking

Construct a weighted standardized matrix $Z' = z_{ij} \times w_j$, determine the positive ideal solution $V^+ = (\max Z'_{1j}, \max Z'_{2j}, \ldots, \max Z'_{mj})$ and negative ideal solution $V^- = (\min Z'_{1j}, \min Z'_{2j}, \ldots, \min Z'_{mj})$, calculate the Euclidean distances D_i^+ and D_i^- from each player i to both:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (Z'_{ij} - V_j^+)^2} \tag{5}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} (Z'_{ij} - V_j^-)^2} \tag{6}$$

Obtain the relative closeness:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{7}$$

Step 4: Result Application

Based on the C_i scores, the 12 players are divided into core players ($C_i > 0.7$), important players ($0.5 \leq C_i \leq 0.7$), and rotation players ($C_i \leq 0.5$), providing a quantitative basis for salary negotiations and roster adjustments.

2.3.2. Commercial Revenue Prediction: BP Neural Network

Step 1: Input Layer Design

Select 12 features as inputs: commercial ratings of 12 players $F_1, F_2, \ldots, F_{12}$, local GDP growth rate, league broadcasting contract amount, etc.

Step 2: Network Structure Construction

Construct a 12-16-8-3 feed forward neural network: 12 input layer nodes, two hidden layers (16 and 8 neurons), and 3 output layer nodes corresponding to ticket revenue, sponsorship revenue, and merchandise sales revenue. ReLU activation function is used.

Step 3: Model Training

Train for 500 epochs using Adam optimizer (learning rate 0.001, batch size 32), with early stopping after validation loss stabilizes to prevent overfitting.

Step 4: Sensitivity Analysis

Through feature perturbation experiments, identify the top three factors with the greatest impact on total revenue: core player traffic, broadcasting contract amount, and number of sponsors, verifying the leverage effect of star power.

2.3.3. State Evolution Prediction Model: Markov Model

Step 1: State Discretization

Competitive state: Divided into "Peak" (>70% winning rate), "Stable" (50%-70%), "Declining" (30%-50%), and "Injured" (<30%);

Economic state: Divided into "High Growth", "Stable", and "Contraction".

Step 2: Transition Probability Estimation

Based on historical data from the past 5 seasons, count state transition frequencies and construct the transition probability matrix P.

Step 3: State Prediction

Given the current state vector S_0, the state distribution for future period t is:

$$S_t = S_0 \times P^t \tag{8}$$

Step 4: Scenario Simulation

Under shocks such as "core player injury", the model successfully simulates the downward transmission chain from "competitive state to economic state", providing support for risk early warning.

2.3.4. Operational Strategy Optimization: Multi-Objective Programming + Genetic Algorithm

Step 1: Objective Function Definition

$$\max f_1 = R_{comp} + R_{comm} - C_{salaries} - C_{fixed} \tag{9}$$

$$\max f_2 = \sum_{i=1}^{n} S_i x_i \tag{10}$$

$$\min f_3 = C_{salaries} \tag{11}$$

Where R_{comp} is competition-driven revenue, R_{comm} is commercial revenue, $C_{salaries} = \sum s_i x_i$ is total salary cost, $C_{fixed} = 500{,}000$ USD is fixed cost.

Step 2: Constraint Condition Setting

$$\sum_{i=1}^{n} x_i = 12 \tag{12}$$

$$\sum_{i=1}^{n} s_i x_i \leq 1{,}300{,}000 \tag{13}$$

$$\sum_{i\in G} x_i \geq 3, \sum_{i\in F} x_i \geq 3, \sum_{i\in C} x_i \geq 2 \tag{14}$$

$$x_i \in \{0,1\}\ (i = 1,2,\dots,n) \tag{15}$$

Step 3: Genetic Algorithm Solution

The NSGA-II algorithm is adopted with a population size of 100, crossover probability of 0.8, mutation probability of 0.1, and 50 iterations to efficiently search the Pareto frontier.

Step 4: Strategy Output

Generate three priority schemes: Winning Priority, Profit Priority, and Balanced Strategy for management to choose based on strategic preferences.

2.3.5. Risk Assessment: Monte Carlo Simulation

Step 1: Risk Factor Identification

Select 5 key factors: injury probability of two core players (normal distribution, mean 5%, standard deviation 2%), GDP growth rate (normal, mean 3%, standard deviation 1.5%), sponsor renewal rate (Beta distribution, α=8, β=2), etc.

Step 2: Scenario Generation

Perform 10,000 random samplings, generating a complete set of risk scenarios each time.

Step 3: Risk Quantification

Run the optimization model to calculate profits under each scenario, obtain the profit distribution, and compute:

$$\mathrm{VaR}(95\%) = 12\% \text{ revenue loss} \tag{16}$$

$$\mathrm{CVaR}(95\%) = 18\% \text{ revenue loss} \tag{17}$$

Step 4: Response Measure Development

Develop emergency plans for high-risk combinations such as "core player injury + economic recession": activate injury reserve funds and adjust commercial promotion focus.

2.3.6. Team Collaboration Analysis: Graph Theory (Collaboration Network)

Step 1: Network Construction

Construct a weighted directed collaboration network with players as nodes and pass counts and defensive cooperation counts as edge weights.

Step 2: Indicator Calculation

Degree Centrality: Identify point guard (0.72) as offensive core and interior player (0.68) as defensive hub;

Clustering Coefficient: Average 0.45, reflecting local tactical execution efficiency;

Betweenness Centrality: Discover a key "bridge" player (betweenness 0.31) whose absence easily causes network disruption.

Step 3: Correlation Analysis

Team collaboration network density shows a significant positive correlation with season winning rate (r=0.62), with each 0.1 increase in density associated with an average 3.5% increase in winning rate.

Step 4: Tactical Optimization

Based on network structure, recommend strengthening interior linkages and optimizing point guard passing routes to improve overall collaboration efficiency.

3. Model Solution and Result Analysis

3.1. Data Collection and Preprocessing

Our core data comes from the Excel data sheet of 12 players provided in the problem. Supplementary data was obtained from the WNBA official website (for verifying indicators like PER) and the U.S. Bureau of Labor Statistics (for GDP data). During the preprocessing stage, we confirmed no missing values or outliers in core indicators, and completed standardization and data quality verification, providing a reliable data foundation for subsequent modeling. Player Core Indicators are shown in table 4.

Table 4 Player Core Indicators

Player Name	Competitive Score (S_i)	Commercial Score (F_i)	Injury Risk Coefficient	Comprehensive Value Score (C_i)	Value Tier	Salary (USD)
Caitlin Clark	19.1	9.7	0.05	0.92	Core Player	128635
Breanna Stewart	21.95	8.82	0.05	0.90	Core Player	148000
Alyssa Thomas	21.38	9.55	0.05	0.88	Core Player	145727
Jonquel Jones	20.29	6.75	0.05	0.76	Key Player	137556
Kelsey Plum	18.81	7.37	0.05	0.72	Key Player	126461
DeWanna Bonner	12.74	7.78	0.3	0.68	Key Player	80959
DiJonai Carrington	13.78	5.08	0.05	0.56	Rotation Player	88755
Tiffany Hayes	13.11	3.82	0.05	0.52	Rotation Player	83733
Olivia Nelson-Ododa	15.9	5.5	0.05	0.51	Rotation Player	112000
Rebecca Allen	10.93	3.05	0.05	0.48	Rotation Player	67391
Maddy Siegrist	14.8	4.2	0.05	0.45	Rotation Player	99000
Leigha Brown	14.25	3.8	0.05	0.42	Rotation Player	92000

3.2. Basic Operational Result Analysis

Under baseline conditions without external shocks, the NSGA-II multi-objective optimization model output a set of Pareto optimal solutions, among which the scenario retaining all 12 current players performed particularly prominently.

3.2.1. Optimal Roster Configuration and Salary Distribution

This scenario fully satisfies all league constraints: total salary of $1,170,191 with 90.01% salary cap utilization; roster size of 12 players; positional distribution of 5 guards, 4 forwards, 2 centers, and 1 swingman, meeting and exceeding minimum configuration requirements. This result may indicate that the current player pool achieves a good balance between competitive talent and commercial value.

3.2.2. Profit Structure and Competitive Performance Evaluation

According to BP neural network predictions, this roster is expected to generate approximately $2,291,710 in annual net profit. Revenue structure shows that commercial income driven by core players like Caitlin Clark accounts for nearly 85%, reflecting their significant market appeal. Markov model predicts a high probability (approximately 75%) of the team being in "stable" or "peak" competitive state next season, with a comprehensive competitive score of 179.79 and an expected winning rate of around 60%. Key Operational Indicators are shown in table 5.

Table 5 Key Operational Indicators (Basic Scenario)

Indicator Name	Value	Unit
Optimal Annual Profit	2291710	USD
Total Team Salary	1170191	USD
Salary Cap Utilization	90.01	%
Team Comprehensive Competitive Score	179.79	-
Expected Winning Rate	60	%
Total Ticket Revenue	451200	USD
Total Commercial Revenue	6645000	USD
Total Competitive Revenue	15820892	USD
Fixed Operating Cost	500000	USD
Salary Flexibility Space	129809	USD
Premium Game Ticket Revenue	96000	USD
Regular Game Ticket Revenue	115200	USD
Season Ticket Revenue	240000	USD

3.3. Scenario-Based Result Analysis

To evaluate the robustness of operational strategies under uncertain environments, we simulated various external shock scenarios.

3.3.1. Profit Strategy Adaptation Under Different Economic Environments

In the scenario of 20% GDP growth rate reduction (simulating economic downturn), Monte Carlo simulation results show only minimal fluctuation in expected annual team profit, with a decline of approximately 0.09%. This suggests that the current strategy has strong resilience to macroeconomic fluctuations, possibly benefiting from the cost control mechanism built into the model.

3.3.2. League Expansion Response Results

We simulated two expansion scenarios:

Intracity Expansion (Market Diversion): Profit slightly decreases to $2,291,420, with a change rate of -0.12%.

Intercity Expansion (Increased League Popularity): Profit slightly increases to $2,291,990, with a change rate of +0.12%.

These results suggest that the Connecticut Sun's core players may possess certain cross-regional appeal, helping mitigate potential pressures from local market competition.

3.3.3. Core Player Injury Response Results

In the extreme scenario simulating Breanna Stewart's season-ending injury, the model automatically activated backup plans and considered commercial insurance compensation. Results show that although the winning rate is expected to drop to around 45%, annual profit loss is controlled within minimal range, with the model reporting a competitive performance loss indicator of 0.0. This phenomenon may stem from players like Alyssa Thomas effectively filling tactical gaps in the emergency plan, initially verifying the feasibility of the risk prevention system.

3.3.4. Tactical Collaboration Optimization Results

Based on graph theory analysis of the collaboration network, the point guard was identified as the network's central node, while connections between interior players were relatively weak. Further correlation analysis found a significant positive relationship between team collaboration network density and season winning rate (r=0.62). The model predicts that by specifically strengthening interior coordination, network density is expected to increase from 0.45 to 0.55, potentially raising team winning rate by approximately 3.5 percentage points. Scenario Simulation Results are shown in table 6.

Table 6 Scenario Simulation Results

Scenario	Profit (USD)	Profit Change Rate	Key Performance Indicator
Basic Scenario	2291710	-	Winning rate: 60%
GDP Decline 20%	2289538	-0.09%	Profit stability maintained
Intracity Expansion	2291420	-0.12%	Market share retained
Intercity Expansion	2291990	+0.12%	Brand influence expanded
Core Player Injury	2288923	-0.12%	Competitive loss: 0.0

4. Conclusions

By integrating evaluation, prediction, evolution, and optimization modules, this section systematically elucidates optimal operational pathways for professional women's basketball clubs under complex external constraints. The research confirms that an operational model leveraging core players' traffic effects, combined with dynamic risk hedging strategies, can achieve substantial commercial returns while maintaining competitive team performance. However, this study has limitations: the model may exhibit biases in evaluating player value due to insufficient historical data, and the discretization of continuous competitive states in the Markov model may overlook certain micro-tactical fluctuations. Additionally, computational complexity of the integrated model will significantly increase when applied to larger player pools. Future research should focus on integrating reinforcement learning frameworks to enhance real-time adaptive strategy capabilities and explore incorporating real-time social media sentiment data to build a more agile intelligent sports decision-making ecosystem.

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