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Co-Optimization Model for Multi-Objective Vehicle Paths in Urban Multilevel Waste Separation Logistics System

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Abstract: Research on Route Optimization and Scheduling of Urban Waste Classification Transportation involves challenges such as route planning, fleet scheduling, and dynamic site selection. This paper constructs a multi-level mathematical model system and proposes hybrid intelligent optimization algorithms, including an improved simulated annealing algorithm, a multi-objective vehicle routing problem model, and a dynamic site selection-routing collaborative optimization model. Aiming at the problems of single-type and multi-type fleet scheduling as well as transfer station site selection, this study aims to achieve cost optimization and route efficiency improvement under dual capacity constraints, and effectively balance computational complexity and solution quality. The research results can provide quantitative decision support for urban environmental sanitation systems and have the expansion potential to fields such as emergency logistics. The innovations are: constructing a stepped model system to achieve full-coverage optimization from single vehicles to multi-fleets and from static to dynamic scenarios; proposing an improved SA algorithm under dual capacity constraints, which enhances convergence performance by innovatively designing dynamic penalty factors.

Keywords: Simulated annealing algorithm; Vehicle path problem; Multi-objective optimization; Stepwise algorithm.

1. Introduction

With the acceleration of urbanization, the production of urban domestic waste is surging year by year, and waste disposal is facing serious challenges. As a key link in the waste management system, waste classification and transportation directly affects transportation costs, environmental benefits and social benefits. Optimizing waste collection and transportation paths not only reduces logistics costs and carbon emissions, but also improves the resource recovery rate and promotes green and low-carbon urban development.

In recent years, research on the optimization of domestic waste collection and transportation routes has shown a trend of shifting from static planning to dynamic response, from single-objective to multi-objective coordination, and from urban areas to rural and suburban extensions. Early studies (2017–2018) focused on the construction of basic models and exploration of classified transportation. For example, Li Wenjie[1] quantified the weight of rural roads (village road weight 0.6396) through the analytic hierarchy process (AHP) and improved the savings algorithm to optimize routes. Jiang Erwei[2] and Wang Chendi[3] used the GIS location-allocation model and queuing theory respectively to optimize the layout of transfer stations and classified transportation modes. They found that although joint transportation reduces the number of vehicles, separate transportation has lower costs. From 2020 to 2021, there were significant algorithm innovations. Zhang Ziwei[4] proposed a multi-recycling station collaborative scheduling model (MSRCP), and Tang Xuan[5] combined gradient boosting regression with the genetic-simulated annealing algorithm (GA-SA) to improve the accuracy of prediction and path optimization. Since 2022, research has deepened dynamic scheduling and low-carbon goals. Li Tingting[6] introduced carbon emission costs and K-means zoning, improved the ant colony algorithm, which reduced the total cost by 35.7% and carbon cost by 25.3%, and developed a hybrid genetic-firefly algorithm to deal with sudden scenarios such as vehicle failures. Lei Xiaozhen[7] improved the genetic algorithm through dynamic adaptive mutation operators, shortening the driving distance. Li Jiasheng[8] constructed a multi-level model of "collection point - transfer station - sorting center - processing plant". The two-stage optimization (Hungarian algorithm + genetic algorithm) showed that the benefits of "step-by-step transfer" are significantly better than the traditional mode. In addition, Jin Yuzhen[9] proposed an AHP-Dijkstra comprehensive evaluation method for rural road networks, taking into account both economic and environmental costs. Chen Li[10] focused on establishing a dynamic path optimization problem model based on traffic flow with time windows and solving it using an improved genetic algorithm.

Research trends involve algorithm hybridization, objective diversification, and technology integration. However, insufficient real-time data integration, poor adaptation to urban-rural differences, and the standardization of carbon accounting remain future breakthrough directions.

This paper is dedicated to solving the key links of waste management in waste classification and transportation, which need to take into account the collection requirements of different waste types, the load and volume limitations of vehicles, the processing capacity of transfer stations and time window constraints, while optimizing transportation costs and carbon emission control. The research focuses on constructing mathematical models to solve three types of problems: first, for single-type waste transportation, aiming at minimizing the total driving distance, determining the number of vehicles, transportation routes and task allocation, and considering multi-trip transportation scenarios and analyzing the limitations of the model in real-world factors

such as traffic congestion; second, expanding to multi-type waste collaborative transportation, based on different vehicle load, volume and transportation cost parameters, establishing a multi-vehicle collaborative model to minimize the total transportation cost, and discussing the impact of driving time constraints on task splitting. The entire research system is committed to improving the efficiency and sustainability of urban waste classification transportation through the integration of multi-dimensional constraints, and providing quantitative decision support for urban environmental governance.

2. Resolution of Garbage Transport Vehicle Routing Problem in PracticalScenarios

2.1. Minimization of Travel Distance for Single Vehicle Routing Scheduling

2.1.1. Model establishment

This section aims to minimize the driving distance of vehicles by finding the optimal path. The goal of model establishment is to find the minimum daily total driving distance under the premise of satisfying all existing conditions. It is necessary to solve the number of transport vehicles, the transport path of each vehicle, and the task allocation. This is a typical vehicle routing problem. Since the same vehicle can make multiple round trips, we construct the model as independent paths where all vehicles start from the processing plant and finally return to the processing plant, and each vehicle can transport the garbage volume corresponding to its path.

Objective function:

$$\min s = \sum_{k_\alpha \in k_{\alpha total}} \sum_{i \in i_{total}} \sum_{j \in i_{total}, i \neq j} x_{ijk_\alpha} d_{ij} \tag{1}$$

Let the right side of the equal sign be U.

constraint conditions:

1. Each collection point is covered by exactly one path.

$$\sum_{k_\alpha \in k_{\alpha total}} y_{ik_\alpha} = 1 \tag{2}$$

2. If a collection point i is associated with a path k_α, there must be a vehicle entering and exiting that garbage collection point.

$$\sum_{j \in E_1, i \neq j} x_{ijk_\alpha} = y_{ik_\alpha} \tag{3}$$

$$\sum_{j \in E_1, j \neq i} x_{jik_\alpha} = y_{ik_\alpha} \tag{4}$$

3. The starting point and the ending point of each path must both be at the treatment plant (0).

$$\sum_{j \in i_{total}} x_{0jk_\alpha} = z_{k_\alpha} \tag{5}$$

$$\sum_{i \in i_{total}} x_{i0k_\alpha} = z_{k_\alpha} \tag{6}$$

4. Vehicle capacity constraint

$$\sum_{i \in i_{total}} w_i y_{ik_\alpha} < Q \tag{7}$$

5. Subtour elimination constraint

To prevent resource waste caused by overlapping paths taken by transport vehicles, we need to analyze and ensure that there are no overlapping segments in the routes operated by the vehicles.

$$A_{ik_\alpha} - A_{jk_\alpha} + i_{total} x_{ijk_\alpha} \leq i_{total} - 1 \tag{8}$$

2.1.2. Model solution

The number of transport vehicles is finally determined. After the model is solved, the number of transport vehicles is equal to the number of paths used.

That is:

$$\sum_{k_\alpha \in k_{\alpha total}} z_{k_\alpha} \tag{9}$$

The transportation route of each vehicle is represented by the road (g, u) we set, and the road must be connected.

The number of tasks required at a collection point is determined by the weight of garbage at that point.

2.2. Solving the path optimization problem with a single vehicle type for practical issues under given conditions

2.2.1. Model establishment

Given that n = 30 and Q = 5, since the problem type is consistent, we can directly apply the aforementioned model for solution.

Thus, the objective function is:

$$\min s = \sum_{k_\alpha}^{30} \sum_{i=0}^{30} \sum_{j=0, j \neq i}^{30} d_{ij} x_{ijk_\alpha} \tag{10}$$

The constraint conditions are completely consistent with the model established above, with only specific values substituted in. Since direct solving may take a long time and the obtained optimal solution may not be accurate, we can compare the simulated annealing algorithm with the savings algorithm to find the optimal solution result and analyze the algorithm complexity.

2.2.2. Model solution

First, with regard to the savings algorithm, through algorithm analysis.

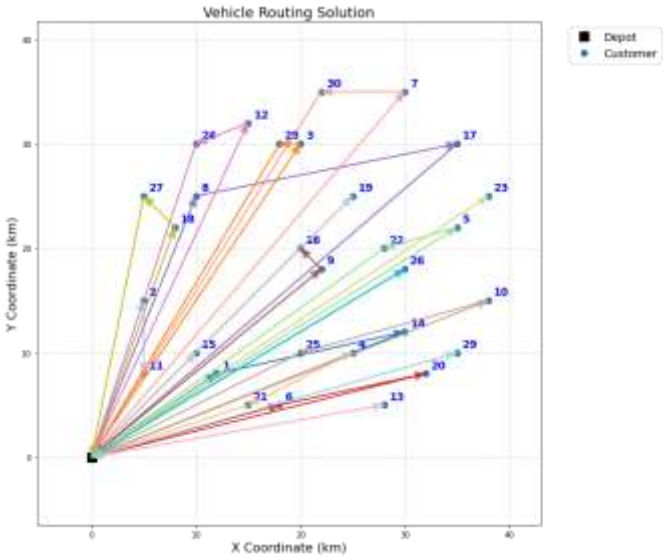


Figure 1 Route selection in the savings algorithm

As can be seen from Figure 1 "Route selection in the savings algorithm", the final calculation results of the savings algorithm indicate that the total driving distance is 1181 km, with the total number of transport vehicles being 17.

Analysis of Simulated Annealing Algorithm:

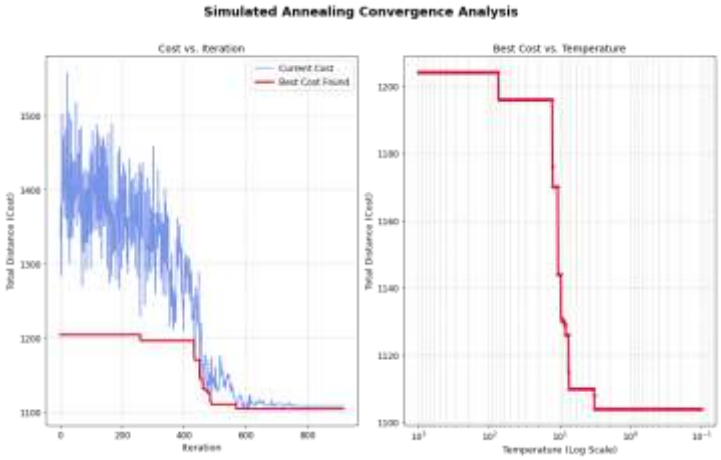


Figure 2 Simulated annealing iteration process

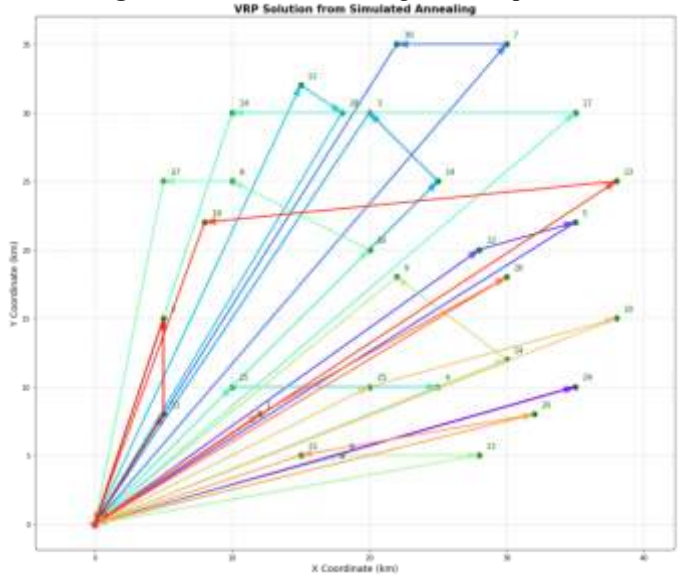


Figure 3 Simulated annealing path selection

According to Figure 2 "Simulated annealing iteration process", we conduct an analysis of the convergence curves. In the case of the simulated annealing algorithm, there are altogether two iteration graphs: the left one depicts the relationship between the number of iterations and the total distance, while the right one illustrates the relationship

between temperature and the total distance. These two graphs intuitively showcase both the iteration process and the value of the final optimal solution.

According to Figure 3 "Simulated annealing path selection", the final total driving distance obtained by the simulated annealing algorithm is 1104 km, with a total of 15 transport vehicles.

Comparison of model complexity: The savings algorithm works by calculating the savings values for all users and then sorting them in descending order of the savings values. Therefore, the time complexity of the savings algorithm is $O(\mathrm{n}^2\log\mathrm{n})$, However, the model complexity of the simulated annealing algorithm depends on the number of iterations and neighborhood operations, so its time complexity is higher. In contrast to time complexity, the space complexity of the simulated annealing algorithm $O(\mathrm{n})$ is significantly lower than that of the savings algorithm $O(\mathrm{n}^2)$. In summary, the savings algorithm has a simple logic and is easy to implement, while the simulated annealing algorithm is more flexible and suitable for complex constraints. Its neighborhood design has strong scalability, but its convergence speed is slow.

2.3. Limitations and Improvements of the Single Vehicle Routing Scheduling Model

The limitations of the model mainly lie in its failure to take into account actual traffic conditions, including traffic congestion and differences in driving speeds between vehicles. The model only considers choosing the shortest path and assumes that all vehicles travel at the same speed, while in real life, both road conditions and vehicle speeds have certain randomness. Meanwhile, our assumption of fixed garbage generation is impossible in reality; furthermore, we have not considered the opening hours of treatment plants. The model adopts ideal results and does not take into account the impact of extreme weather or unexpected situations on transportation. Moreover, in the process of solving the target, we have simplified the actual road network. Since there may be asymmetric constraints such as one-way streets and restricted driving periods, they are not fully considered in the model.

To address the aforementioned limitations of the model, we have chosen to introduce the time factor into the model for improvement. In order to make it more realistic and achieve practical application effects, the constraint of time windows has a significant impact. During the travel of transport vehicles, differences in their driving speeds will result in changes in travel time. Therefore, we can transform the fixed distance d_{ij} into an expression with time-dependent effects T_t, We can find data to conduct an analysis of historical traffic, so as to predict driving speeds and travel times during different time periods. Furthermore, our objective function will shift from finding the minimum driving distance to finding the minimum travel time.

We will refine the collection points required by the model and set a time range from t_1 to t_2 during which transport vehicles are permitted to carry out transportation. Among them, t_1 is the earliest opening time, and t_2 is the latest closing time. If there is a transport vehicle coming to transport garbage, the work will start at the arrival time of the transport vehicle, denoted as T_{t_1} ,There is $T_{t_1} \geq t_1$,End time T_{t_2} ,There is $T_{t_2} \leq t_2$. Meanwhile, we take into account that the processing plant also has corresponding working hours, and set its working window from t_{chu} to t_{li} , Vehicles transporting garbage must operate strictly within the specified time.

This article will adjust the model and introduce a new decision variable $T_{k_\alpha i}$. It represents the time of arrival at the i-th collection point, and the newly added constraint is $T_{k_\alpha} \geq T_{k_\alpha} + T_t T_{k_\alpha}$. Meanwhile, the total working hours of the vehicle must not exceed the maximum duration.

3. Optimization under the collaboration of multiple vehicles and load constraints

3.1. The collaborative transportation of multi-type vehicles and the optimization under multiple constraints.

3.1.1. Model establishment

For multi-vehicle scenarios, different types of garbage are involved. That is, different garbage collection points may generate different kinds of garbage, and each kind of garbage corresponds to a specific type of transport vehicle. Each vehicle type has its respective volume and load constraints. Our goal is to establish a multi-vehicle collaborative transportation model that minimizes the total daily transportation cost. Since different types of vehicles can be scheduled independently without affecting other types, we can, under the premise of single-vehicle operations, transform the optimization problem of multi-vehicle collaboration into k independent vehicle routing problems.

Objective function:

$$\min \mathrm{S} = \sum_{k_\alpha \in k} s_k \tag{10}$$

$$s_k = C_k * U \tag{11}$$

Constraint conditions:

1. Ensure that the k-th type of garbage at each collection point is collected by a dedicated vehicle, and it is collected exactly once.

$$\sum_{k_\alpha \in k} y_{ikk_\alpha} = 1 \tag{12}$$

2. If collection point i is associated with route k, there must be vehicles entering and exiting this garbage collection point.

$$\sum_{j \in E_1, j \neq i} x_{jik_\alpha k} = y_{ik_\alpha k} \tag{13}$$

$$\sum_{j \in E_1, j \neq i} x_{ijk_\alpha k} = y_{ik_\alpha k} \tag{14}$$

3. Both the starting point and the ending point of the route must be at the processing plant (0).

$$\sum_{j \in i_{total}} x_{0jk_\alpha k} = z_{k_\alpha k} \tag{15}$$

$$\sum_{i \in i_{total}} x_{i0k_\alpha k} = z_{k_\alpha k} \tag{16}$$

4. Dual constraints on vehicle load and volume.

Load constraint:

$$\sum_{i \in i_{total}} w_{ik} y_{ikk_\alpha} \le Q_k * z_{k_\alpha k} \tag{17}$$

Volume constraint:

$$\sum_{i \in i_{total}} V_{ik} y_{ikk_\alpha} \le V_k * z_{kk_\alpha} \tag{18}$$

5. Subtour constraint:

$$A_{ikk_\alpha} - A_{jkk_\alpha} + i_{total} x_{ijkk_\alpha} \le i_{total} - 1 \tag{19}$$

3.1.2. Model solution

Essentially, this model is an integration of generalized models for multiple single-vehicle transportation scenarios. The final result is obtained by summing up the calculation results of multiple minimized transportation costs, which constitutes the objective of the model we have constructed.

3.2. For the practical solution to the optimization problem involving multi-type vehicles and constraint conditions in specific schemes.

3.2.1. Model establishment

The article provides detailed information on the number of collection points and types of garbage, as well as the load limits, volume limits of vehicles, and the cost per unit transportation distance for different vehicles. We need to extend the single-vehicle algorithm to more complex application scenarios. As we have already presented a generalized model solution for this problem in the preceding text, we will now substitute specific data into the model for solving, and adjust to find the optimal solution.

Objective function:

$$\min S = \sum_{k=1}^{4} C_k \sum_{k_\alpha \in k} \sum_{i=0}^{30} \sum_{j=0, j \ne i}^{30} d_{ij} x_{ijkk_\alpha} \tag{21}$$

The constraint conditions remain consistent with those mentioned above, and only the numerical values need to be substituted.

Since two algorithms were employed for the single-vehicle scenario, and through solving the problem, we found that the simulated annealing algorithm is more reliable than the savings algorithm, this section will extend the application of the simulated annealing algorithm.

To extend the algorithm so that it is applicable to the scenario of this problem, in addition to the data related to a single vehicle itself, we need to process new data. This includes identifying the actual sources of each type of garbage, accurately applying the number of vehicles for class-k garbage and their limiting parameters, and calculating the weight of the garbage based on its volume and density.

In the multi-vehicle collaborative garbage collection problem, the simulated annealing algorithm achieves global optimization under complex constraints through cross-station path representation, neighborhood operations, and objective functions. This problem requires assigning paths covering all garbage points to multiple vehicles, with the goals of minimizing the total travel distance, balancing vehicle loads, and meeting capacity constraints.

The algorithm adopts grouped path coding, explores the solution space through neighborhood operations such as cross-vehicle node migration, path merging, or splitting, and introduces penalty terms for load differences and capacity violations in the objective function. The path of each vehicle must satisfy the sub-circuit constraint, and at the same time, the Metropolis criterion is used to probabilistically accept worse solutions to jump out of the local optimal solution.

Perform the above algorithmic operations on all types of garbage, and obtain the optimal solution for this section by summing up the results.

The main changes in the model's constraint conditions are as follows:

1. The previous text focused on the scheduling problem for a single vehicle type, while this section discusses the assignment for multiple vehicle types. Therefore, when establishing the constraint conditions, the independent situations of k types must be considered.
2. Multiple vehicles are subject to dual constraint restrictions, including load limits and volume limits.

Regarding the generation of different types of garbage, we need to clarify the number and locations of collection points where such garbage is generated.

3.2.2. Model solution

First, process the existing data. We intuitively express the characteristics of the data by using a heat map for both the coordinate map of node distribution and the distance matrix between nodes. The node distribution map on the left intuitively presents the geographical coordinate distribution of processing plants (in red) and collection points (in blue), helping to identify node aggregation, sparse areas, or abnormal locations, thus providing spatial references for path planning. The distance matrix heat map on the right reflects the Euclidean distance between any two nodes through color depth, revealing global distance patterns and assisting in quickly locating adjacent node combinations to optimize paths. The combination of the two can intuitively analyze the spatial relationship between nodes, verify data quality, and provide visual decision support for vehicle path planning.

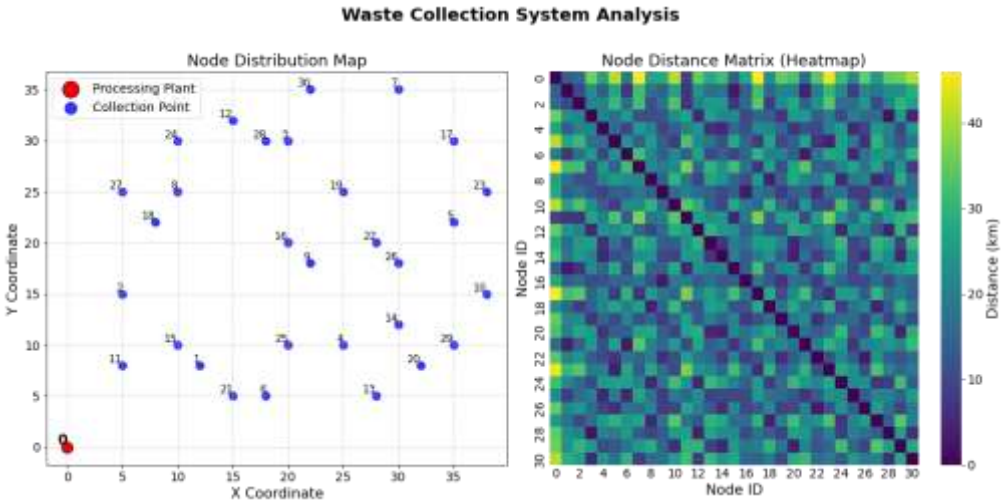


Figure 4 Coordinate distribution and node distance

According to Figure 4 "Coordinate distribution and node distance", we can intuitively analyze the coordinate distribution of garbage collection and the node distances.

In this section, the simulated annealing algorithm with better output performance is utilized to solve this practical problem.

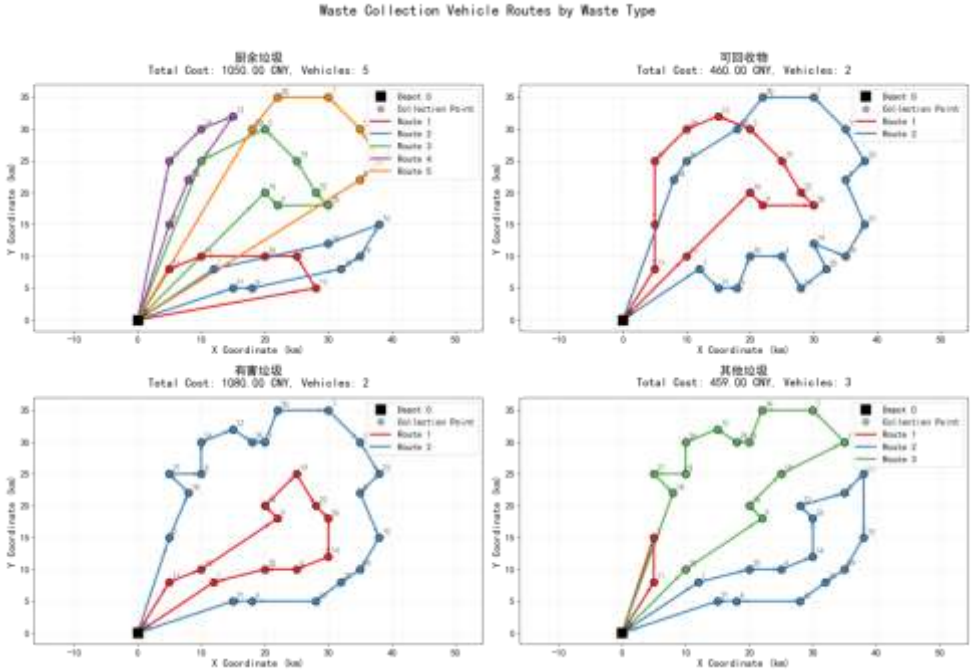


Figure 5 Path selection

According to Figure 5 “Path selection”, analyze costs, vehicle numbers and routes of different waste types to understand the layout logic and efficiency of classified collection. The final minimum daily transportation cost is 3,049 yuan, with a total of 12 vehicles used.

3.3. Model Improvement for Multi-Type Vehicle Routing Optimization

On the basis of the original model, a new constraint of the maximum daily driving time of vehicles is added, and we need to modify the model to ensure that this constraint is satisfied. Through analysis, it is determined that each of the k types of vehicles corresponding to k types of garbage has its own maximum daily driving time $T_{\max(k)}$, To simplify the problem, we assume that the maximum daily driving time is the same for all k types of vehicles. To incorporate this constraint, we need to calculate the total time for each route, where the total time of a route includes both the total driving time and the total working time.

Unit road driving time:

$$T_{k_\alpha} = d_{k_\alpha} / \bar{v} \tag{20}$$

Total driving time:

$$T_{k_\alpha total} = \sum_{k_\alpha \in k_{k_\alpha}} T_{k_\alpha} \tag{21}$$

The time spent loading or unloading at transfer station j is denoted as t_{jk}, and the time spent collecting garbage at collection points is denoted as t_{ik}. Then, the total working time is expressed as

$$T_{total} = \sum_{k \in k_\alpha} (\mathrm{t}_{ik} + \mathrm{t}_{jk}) \tag{22}$$

Therefore, the maximum driving time of the vehicle on each road is

$$T_{total} = T_{k_a total} + T_{total} \tag{23}$$

This newly added constraint will be directly incorporated into the model's solution process and influence the final results. It does not affect other constraints we have established, thus making the model closer to real-life scenarios.

After adding this constraint, the number of vehicles may increase. If the operation time of one or more routes exceeds the upper limit, making transportation impossible, the route needs to be split into several sub-routes. For example, a route with a total distance of 300 km and 9 collection points to visit exceeds the maximum driving time T = 6 hours (based on calculated transportation time). In such cases, the route can be split into sub-routes: sub-route A covers 170 km and visits 4 collection points, while sub-route B covers 130 km and visits 5 collection points, thus meeting the time constraint.

4. Conclusions

This study focuses on the optimization and scheduling of urban waste classification and transportation routes. It constructs a multi-level mathematical model system and designs a hybrid intelligent optimization algorithm, forming a solution with engineering application value.

For single-vehicle path planning, a Vehicle Routing Problem (VRP) model with multiple backhaul constraints is established. Comparative experiments between the savings algorithm and the improved Simulated Annealing (SA) algorithm, verified by a 30-node example, show that the optimal solution of the improved SA algorithm (with a total mileage of 1,104 kilometers and 15 paths) is superior to traditional methods. Moreover, with a time complexity of $O(n^2)$, it effectively balances computational efficiency and solution quality. In view of the model's limitations, a method for constructing multi-objective functions embedded with dynamic traffic flow parameters and an improvement direction for the random demand response mechanism are proposed.

To meet the scheduling needs of heterogeneous fleets with multiple vehicle types, a multi-objective VRP model integrating dual weight and volume capacity constraints is developed. Vehicles are allocated by waste type: Type 1 vehicles (5 routes) for kitchen waste, Type 2 (2 routes) for recyclables, Type 3 (2 routes) for hazardous waste, and Type 4 (3 routes) for other waste. The average daily total cost is optimized to 3,049 yuan. The model quantitatively analyzes the sensitivity of time window constraints to path planning, enabling dynamic scheduling. It is found that dual capacity constraints significantly restrict vehicle utilization, requiring multi-objective trade-offs to achieve Pareto optimization. Future research can explore the introduction of reinforcement learning and dynamic programming methods to further enhance the practicality of the model.

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