

Construction and Practical Research on Teaching Pathways for Problem Posing in Junior High School Mathematics Based on Polya's "How to Solve It" Philosophy

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Abstract: With the 2022 Compulsory Education Mathematics Curriculum Standards elevating "problem posing" as a core competency, fostering this skill has become crucial. However, junior high mathematics teaching often prioritizes problem solving over problem posing, limiting students' creative thinking. Based on Polya's heuristic ideas in *How to Solve It*, this study constructs a systematic teaching pathway for problem posing. By analyzing key elements in Polya's philosophy—such as analogy, rephrasing, decomposition, and specialization—and integrating the "What-If-Not" strategy, a four-stage teaching model is developed: overall questioning, systematic inquiry, deep exploration, and practical innovation. Using the "similar triangles" unit as a case study, the paper details instructional design and examines the use of AI tools in selecting student-generated problems. This approach offers an innovative, operable framework to enhance question-posing ability, supporting the shift toward student-centered mathematics teaching focused on core competencies.

Keywords: Polya, Problem Posing, Junior High School Mathematics, Teaching Pathway, Core Competencies.

1. Introduction

With the deep implementation of the Compulsory Education Mathematics Curriculum Standards (2022 Edition), the ability to "pose problems" has been explicitly identified as a crucial component of core mathematical literacy. The standards emphasize that students should be able to discover and pose problems in real-life contexts and comprehensively apply mathematical knowledge to analyze and solve them [1]. This emphasis is consistent with the international trend in mathematics education. For example, Cai et al. analyzed the 2022 Chinese mathematics curriculum standards and found that problem posing is mentioned more than 80 times, highlighting its role as a cognitive activity, an instructional goal, and an instructional approach [2]. Similarly, Silver and Li noted in their cross-national review that although problem posing is increasingly emphasized in policy documents—such as the NCTM standards in the United States—its actual integration into textbooks and classroom instruction remains limited across many educational systems [3].

However, the current reality of classroom teaching is far from optimistic. In today's secondary mathematics classrooms, there is still a prevalent phenomenon of emphasizing problem-solving while neglecting problem-posing, and students' initiative and creativity in asking questions have not been fully developed or effectively guided [4–9]. For instance, Muirhead et al. observed in U.S. middle school classrooms that even when using curricula such as *Illustrative Mathematics* that include some problem-posing tasks, teachers often struggle to implement them effectively without additional support [10]. In China, Guo Yufeng et al. also pointed out that although problem posing is included in curriculum standards, its representation in textbooks remains sparse and unevenly distributed across content areas, with

most tasks appearing in exercises rather than integrated into main content [11]. This situation clearly contrasts with the student-centered teaching philosophy advocated by the new curriculum standards and urgently requires systematic instructional intervention [12].

The heuristic teaching approach proposed by the Hungarian mathematician G. Polya in his classic work *How to Solve It* provides a solid theoretical foundation for cultivating students' ability to pose problems. The series of heuristic inquiries he advocates during problem-solving—such as "Do you know a similar problem?" and "Can you restate the problem?"—essentially guide students to stimulate mathematical thinking and curiosity through self-questioning [13]. Polya's educational philosophy not only focuses on the final solution to problems but also, on a deeper level, values the process of question generation, embodying a teaching philosophy in which "problem-solving" and "problem-posing" complement each other [14]. Therefore, creatively applying Polya's ideas to teaching problem posing in junior high school mathematics has significant practical and theoretical value.

This study aims to use Polya's mathematical philosophy as its theoretical framework, address the practical challenges in teaching students to pose problems, and explore systematic pathways and methods for effectively cultivating students' problem-posing abilities in junior high mathematics classrooms. Taking the core chapter "Similar Triangles" as an example, the study designs a complete instructional pathway and demonstrates practical teaching strategies, with the goal of providing frontline mathematics teachers with actionable and replicable instructional models. This effort is intended to promote a profound transformation in mathematics classroom teaching and support the successful implementation of curriculum objectives oriented toward core mathematical literacy.

2. Analysis of the Elements of Problem Posing in Polya's Problem-Solving Ideas

In “How to Solve It,” George Polya systematically elaborates on heuristic thinking, with its core being the guidance of thought through a series of self-posed questions. This is not only a strategy for solving problems, but also the very source of problem posing. These strategies collectively form an actionable thinking scaffold for formulating problems. In this study, they are summarized and translated into the following seven core elements.

2.1. Posing Related Questions through Analogy and Association

Polya emphasizes that when faced with unfamiliar problems, one should actively recall: “Do you know a problem related to this one?” This idea encourages students to build connections between new and existing knowledge. For example, when learning about “similar triangles,” students can be guided to compare with the criteria and properties of “congruent triangles” and ask: “Is congruence a special case of similarity where the sides are in a 1:1 ratio?” “Can the criteria for congruence be ‘relaxed’ to apply to similarity?” Analogical association reduces students’ cognitive load when encountering new knowledge and serves as an effective means of generating new problems.

2.2. Posing New Questions through Rephrasing

By changing the language, symbols, or form of a problem, new dimensions can be revealed. For example, turning the algebraic equation “ $2x+3=7$ ” into “What number, when multiplied by 2 and then increased by 3, equals 7?” helps students grasp the essence of inverse operations. In geometry, restating definitions—such as defining a “parabola” as “the locus of points equidistant from a fixed point and a fixed line”—can give rise to numerous new questions related to distance and loci.

2.3. Posing Subproblems by Decomposing Conditions

When facing complex problems, Polya recommends “separating the different parts of the conditions” and analyzing the independent and interrelated effects of each condition to pose a series of subproblems. For instance, in proving geometric theorems like the congruence of two triangles, students can be guided to decompose the conditions and ask independently: “Is it enough if only the sides are equal?” “What if only the angles are equal?” “How can one side and two angles be combined?” Through such exploration, students can gradually approach the complete congruence criteria.

2.4. Posing Questions through Specialization and Generalization

Specialization (considering extreme or particular cases) and generalization (extending to broader cases) are crucial methods in mathematical discovery. For example, when studying the properties of general triangles, one may begin with special cases such as equilateral or right triangles and ask, “Can the results for equilateral triangles be generalized to isosceles triangles?” When learning about quadratic functions $y = ax^2 + bx + c$, one can start with the specific form $y =$

ax^2 and progressively generalize, with each step generating new questions.

2.5. Posing Questions by Introducing Auxiliary Elements

Polya points out that applying known theorems often requires introducing auxiliary elements—a process that itself constitutes problem posing. For example, during a geometric proof, when progress stalls, students can ask: “Can I draw an auxiliary line (such as a median or altitude) to create a familiar figure (like a right triangle)?” “Can I introduce an auxiliary unknown to simplify algebraic relationships?” Every introduction of an auxiliary element stems from a new question raised to solve the original problem.

2.6. Posing Questions through Verification and Reflection

Reflection after problem solving is the essence of Polya’s thinking. He advocates deepening understanding by asking questions like “Can you check this result?” “Can you derive this result in another way?” In teaching, substituting the solution back into the equation is not only validation but also prompts reflection: “What does this solution mean?” “How would changing a constant in the problem affect the solution?” Such reflective questioning reinforces knowledge and generates deeper, more meaningful questions.

2.7. Posing Questions through Observing Diagrams or Symbols

“Draw a diagram” or “introduce suitable notation” are visualization strategies advocated by Polya. Diagrams and symbols are not only tools for problem solving but also sources of inspiration for posing questions. For instance, observing the graph of a function can prompt questions about monotonicity, extrema, or symmetry; using a Venn diagram can lead to questions about the relations of set intersection, union, and complement. Guiding students to extract information and raise questions from diagrams and symbols is key to cultivating mathematical intuition and symbolic sense.

3. Constructing a Teaching Pathway for Problem Posing in Junior High School Mathematics Based on Polya's Ideas: Taking “Similar Triangles” as an Example

This chapter takes the core content of junior high school geometry, “similar triangles,” as an example to construct a four-stage teaching pathway for problem posing that integrates Polya’s ideas with the “What-If-Not” method. The aim is to guide students from being mere recipients of knowledge to becoming discoverers, proposers, and creators of problems. The entire teaching path begins with a contextual introduction and holistic problem design, adhering to Polya’s concept of “returning to definitions” to stimulate students’ macro-level thinking about the essence of similar triangles. For instance, teaching can be introduced with the story of the ancient Greek mathematician Thales measuring the height of a pyramid, raising the central question: “Why does a stable proportional relationship exist between two seemingly unrelated objects — the shadow and the pyramid?” Subsequently, students are guided to pose macro-level

questions from four perspectives: “time” (the origin and evolution of the concept of similarity), “space” (how different cultures understand similarity), “applications” (phenomena of similarity in architecture, art, and nature), and “internal mathematics” (the relationships among similarity, congruence, proportion, and area). In this way, similar triangles are placed within a broad and vivid network of knowledge.

Based on this macro understanding, teaching proceeds to the second stage: systematic questioning and problem generation. This phase utilizes Polya’s questioning chain and the What-If-Not method to guide students in an in-depth analysis of the criteria and properties of similar triangles. Teachers can provide a standard proposition, such as “If two triangles have equal corresponding angles, then they are similar,” and prompt students to engage in activities like “restating” or “drawing analogies.” More importantly, by systematically negating the conditions and conclusions of the proposition, a series of exploratory questions can be generated. For example, by negating “corresponding angles are equal,” one might ask: “If corresponding angles are not equal but all three sides are proportional, are the triangles still similar?” or “If this occurs on a spherical surface, does the conclusion still hold?” Through this process, students can independently “rediscover” the criteria for determining similar triangles and understand their limitations. The specific process of problem generation can be referenced in the table 1.

Table 1. Examples of Problem Generation in Exploring the Properties of Similar Triangles

Original Proposition	Generated Questions
Corresponding angles are equal	If only two sets of angles are equal, are the triangles necessarily similar?
Corresponding sides are proportional	If three sides are proportional but the angles are not equal, is this possible?
Similar triangles in a plane	Is there a concept of similar triangles on a sphere or curved surface?
Side length ratios are equal	If the side length ratio is an irrational number, does similarity still hold?
General triangles	Do the similarity criteria apply to right triangles? To isosceles triangles?

Once students have generated a large number of questions, the teaching enters the third stage: AI-assisted screening and in-depth inquiry. To address the challenges of filtering questions, teachers can introduce artificial intelligence (AI) tools, following Polya’s idea of “checking results,” to create an evaluation framework encompassing dimensions such as “originality,” “mathematical depth,” “verifiability,” and “interdisciplinary potential.” This allows for efficient selection of questions with significant investigative value. For example, teachers may instruct AI tools to prioritize students’ questions according to pre-set criteria, assisting in organizing students to conduct three types of investigative activities: “basic validation,” “extended exploration,” and “interdisciplinary application.”

Finally, the teaching pathway culminates in the stage of creative expression and presentation of results. Polya believed that the ultimate goal of learning mathematics is application, so this final stage aims to transform students’ investigative findings into practical innovations. Under teacher guidance, students may choose creative tasks such as “mathematical modeling” (e.g., designing a method for measuring the height of the school flagpole) or “artistic design” (e.g., using similarity transformations to create Escher-style tessellations). In the presentation phase, students not only display their

works but, more importantly, apply Polya’s strategy of “review and reflection” to recap the entire process from posing to solving problems. This reinforces students’ awareness of problem posing and helps them develop sustainable habits of mathematical thinking.

4. Conclusions

This study is grounded in Polya’s “How to Solve It” philosophy as its theoretical foundation, and systematically constructs a teaching pathway for junior high school mathematics that establishes a two-way thinking channel between “problem solving” and “question posing.” Innovatively integrating Polya’s self-questioning strategies with the “negation hypothesis method,” the research develops a clear, highly operable framework for question posing. This framework encompasses the entire process—from introducing a macro context, systematically raising doubts, AI-assisted screening, to interdisciplinary creative expression. Through practical application in the chapter on “Similar Triangles,” the study demonstrates how this pathway guides students from passively receiving knowledge to actively constructing meaning, achieving a transformation from “teacher-centered” to “student-centered” learning, and from “knowledge transmission” to “thinking stimulation.”

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