Performance Verification of BFS For Unweighted Maze Solving: A Comparative Analysis with DFS and A* Via Ocaml Implementation

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Abstract. Maze solving is a key pathfinding problem with applications in robotic navigation and game Artificial Intelligence (AI). This study focuses on verifying Breadth-First Search (BFS)'s advantages in unweighted mazes, comparing it with Depth-First Search (DFS) and the A* algorithm via OCaml implementation. Five typical mazes (simple, complex, enclosed, empty, duplicate) were designed, with performance evaluated using metrics: path length, traversed nodes, solving time, and shortest path guarantee. Results show BFS consistently guarantees the shortest path (unlike DFS, which falls into depth traps) and has stable efficiency—outperforming A* in small/simple mazes by avoiding heuristic calculation overhead. BFS's simplicity (relying on basic data structures) and strong adaptability across maze types confirm it as the ideal choice for unweighted maze solving. These findings also highlight its educational value as a baseline algorithm for introducing pathfinding concepts. Future work may extend to weighted mazes, heuristic refinements, and hybrid algorithms combining BFS with optimization strategies.

Keywords: DFS; BFS; A*; Maze algorithm.

1. Introduction

Maze solving, a classic pathfinding problem in computer science, is pivotal for algorithm education and practical applications like robotic navigation, game AI, and logistics planning. Its core goal is finding an obstacle-avoiding path between start and end points in a grid—algorithm choice directly impacts path optimality, efficiency, and applicability. Among solutions, Breadth-First Search (BFS) stands out as the ideal choice for unweighted mazes (where all passable paths have consistent weights) due to its inherent guarantee of finding the shortest path [1].

Rooted in early 20th-century graph theory, the foundational ideas of these pathfinding algorithms evolved alongside the field's development. Depth-First Search (DFS), conceptually traced to the 19th-century work of mathematicians on graph traversal, was formalized in the 1950s as a straightforward method for exploring connected components, leveraging stack-based logic to dive deep into graph branches [2]. BFS, meanwhile, emerged around the same period, with its level-order traversal first systematically described in algorithms for shortest-path problems in unweighted graphs—its ability to guarantee optimal paths made it a staple in network routing and grid navigation early on [1]. The A* algorithm, a more recent advancement, was proposed by Peter Hart, Nils Nilsson, and Bertram Raphael in 1968, introducing heuristic guidance to balance BFS's optimality and DFS's directional focus, revolutionizing pathfinding in fields like robotics and game development by reducing unnecessary node traversal [3].

In contrast, DFS is simple to implement but prone to getting stuck in deep, non-optimal paths; while DFS efficiently explores graph branches using a stack (or recursion) and minimizes memory usage by avoiding storage of entire levels, it lacks BFS's shortest-path guarantee and often backtracks excessively in complex grids [4]. The A* algorithm, though more efficient than BFS in large grids when paired with admissible heuristics (e.g., Manhattan distance), requires careful design of cost functions and maintains node order via priority queues—adding complexity that outweighs its benefits in small or simple mazes [3]. This paper uses an OCaml-based maze solver as a vehicle to analyze BFS's code logic and advantageous characteristics, compares it with DFS and

A* across multiple dimensions to highlight BFS's irreplaceability in unweighted maze scenarios, and showcases the type safety and modular advantages of the OCaml language in algorithm implementation [5].

To verify BFS's advantages, this paper selects five typical maze cases for testing: a simple maze with sparse obstacles, a complex maze with narrow passages, an enclosed maze with a pre-built complete outer wall, an empty maze (with no obstacles), and a duplicate maze with the same structure as the first case. These cases cover BFS's common application scenarios and edge cases, enabling a thorough comparison of how BFS, DFS, and A* perform in different environments. For example, in a complex maze with narrow passages, DFS may backtrack repeatedly due to prioritizing dead-end exploration, leading to far more nodes traversed than BFS [2]; A* can reduce the number of traversed nodes via heuristic functions but requires extra Manhattan distance calculations, and in scenarios where passages only allow a single path, its advantages are negligible—even increasing complexity due to priority queue operations [3]. BFS, by contrast, can directly find the shortest path through level-order traversal without backtracking, achieving a balance between efficiency and simplicity [1].

Subsequent sections of this paper will elaborate on the details of BFS's code implementation, compare the performance of DFS and A* in the five typical cases to highlight BFS's strengths, discuss BFS's limitations and potential improvement directions, and finally summarize BFS's core value in solving unweighted mazes.

2. Method

This section elaborates on three classic pathfinding algorithms—DFS, the A* algorithm, and BFS—focusing on their core concepts, principles, implementation workflows, and operational details. With BFS as the core of the study, its technical implementation logic and design ideas for adapting to maze solving are explained in depth.

2.1. DFS

DFS follows the concept of "depth-prioritized traversal": starting from the maze's start point, it continuously explores in a single direction (e.g., up, down, left, right) until it encounters an obstacle or a visited node (a "dead end"), then backtracks to the nearest branch that has not been fully traversed and repeats the exploration process. Its principle is based on a Last-In-First-Out (LIFO) mechanism, typically implemented using an explicit stack or implicit recursion [2]. It only retains the "path from the start point to the current node" and does not need to store nodes of the entire current level, resulting in low memory usage [4]. During implementation, a boolean matrix (used to mark whether a node has been visited to avoid cyclic traversal) and a direction set (defining four possible movement directions) are first initialized. The start point is pushed onto the stack (or recursion is initiated) and marked as visited. After popping a node from the stack, the coordinates of its adjacent nodes are calculated and validated: the nodes must be within the maze boundaries, passable (no obstacles), and unvisited. Valid nodes are pushed onto the stack and marked as visited, while invalid nodes trigger backtracking to try the next direction. However, DFS has significant limitations: it cannot guarantee the shortest path—if a deep dead end is chosen at a fork, the path found after backtracking may be much longer than a shallower path; furthermore, it is prone to falling into "depth traps" in complex mazes, traversing a large number of irrelevant nodes before returning to the correct direction, which greatly reduces solution efficiency [1].

2.2. A* Algorithm

The A* algorithm is a "heuristic-guided" pathfinding method that combines the optimality of BFS and the directionality of DFS. Its core lies in prioritizing the exploration of nodes closer to the end point through a comprehensive cost function. The cost function is defined as $f(n) = g(n) + h(n)$: $\mathrm{g(n)}$ represents the actual distance from the start point to the current node n (obtained by step

counting), and $h(n)$ represents the estimated distance from n to the end point (Manhattan distance—sum of the absolute differences of horizontal and vertical coordinates—or Euclidean distance is commonly used in grid mazes) [3]. The algorithm expands nodes in ascending order of $f(n)$, and if $h(n)$ satisfies the "admissibility" condition (the estimated distance does not exceed the actual distance), it can guarantee the shortest path [1]. Implementation requires a priority queue (sorted by $f(n)$): the start point is enqueued with $g(n)$ =0, $h(n)$ =heuristic value, and $f(n) = g(n) + h(n)$. Meanwhile, two auxiliary matrices are initialized—one records the minimum $g(n)$ value for each node (to avoid reprocessing high-cost paths), and the other tracks the predecessor relationship of nodes (for subsequent path reconstruction) [3]. Each time, the node with the smallest $f(n)$ is extracted from the queue; if it is the end point, the algorithm terminates and reconstructs the path; if not, it iterates over four directions, verifies the boundaries, passability, and necessity of $g(n)$ updates for adjacent nodes. Nodes that meet the criteria have their $f(n)$ recalculated before being enqueued. However, A* relies on prior knowledge of the maze (e.g., the end point coordinates must be known, and the grid structure must be regular). Improper design of $h(n)$ (e.g., an excessively high estimated distance) will lose the guarantee of optimality, and its efficiency may even be lower than that of BFS [6]. Additionally, the maintenance of the priority queue increases implementation complexity, and in small or simple mazes, the additional overhead of heuristic calculation may offset its efficiency advantages [4].

2.3. BFS

As the core of this study, BFS adopts the concept of "level-order traversal": starting from the start point, it first traverses all nodes at a distance of 1 (the first level), then all nodes at a distance of 2 (the second level), forming a "layer-by-layer diffusion" pattern [1]. Its principle is based on a First-In-First-Out (FIFO) queue, which ensures that "when the end point is first encountered, the path taken is the shortest path"—because all shorter paths will be explored in earlier levels [1, 2]. The implementation of BFS revolves around three core components: an optimized queue (by separating front and rear lists, the average time complexity of enqueue and dequeue operations is optimized to O (1), avoiding the efficiency bottleneck of traditional single-list queues) [2], a visited marking matrix (recording whether nodes have been traversed to prevent repeated exploration), and a predecessor node matrix (recording the source node of each node for path reconstruction) [5]. The specific workflow is as follows: after initializing the queue, the start point is enqueued and marked as visited; when entering the loop, the current node is extracted from the queue. If it is the end point, the predecessor node matrix is used to backtrack from the end point to the start point, and the path is organized in the order of "end point → start point" into a complete path of "start point → end point"; if it is not the end point, it iterates over four directions, calculates the coordinates of adjacent nodes, and verifies them (being within boundaries, passable, and unvisited). Nodes that pass the verification are marked as visited, their predecessor nodes are recorded, and they are enqueued to enter the next round of traversal. Compared with DFS (which cannot guarantee the shortest path) and A* (which requires complex heuristic design and relies on prior knowledge), BFS has significant advantages: it does not require prior information about the maze and can stably traverse all reachable nodes regardless of whether the obstacle distribution is regular [1]; it achieves stable efficiency with the optimized queue and only relies on the queue and two auxiliary matrices, featuring concise logic. It balances efficiency and ease of use in small and medium-sized mazes, making it an ideal choice for unweighted maze solving [5].

3. Results and Discussion

This section focuses on the experimental design, result presentation, and in-depth analysis of three path-finding algorithms in unweighted maze scenarios, with an emphasis on verifying the advantages of BFS (Breadth-First Search) in terms of shortest path guarantee, efficiency stability,

and environmental adaptability. Results are presented through quantitative data (tables) and trend analysis (charts).

3.1. Experimental Setup

To comprehensively evaluate the performance differences between the core algorithm BFS and the other two algorithms (DFS and A*), five typical unweighted mazes were designed, covering common application scenarios and boundary cases [7]. Below are the design parameters of the mazes and the definition of evaluation indicators:

The five mazes vary in grid size, obstacle density, and structure to simulate different types of unweighted mazes:

1) Simple Maze: 20×20 grid, obstacle density of 0.3 (sparse obstacles, no narrow passages);
2) Complex Maze: 40×40 grid, obstacle density of 0.5 (dense obstacles with narrow passages, prone to triggering "depth traps");
3) Enclosed Maze: 30×30 grid, full outer surrounding walls + internal obstacle density of 0.4 (limited space, requiring traversal of all reachable nodes);
4) Empty Maze: 50×50 grid, obstacle density of 0 (no obstacles, used to test algorithm efficiency in large empty spaces);
5) Repetitive Maze: 20×20 grid, identical structure to the simple maze (used to verify the repeatability of algorithm results).

Four core indicators were used to compare algorithm performance [8]:

1) Path Length (L): Euclidean distance of the final path (to verify the ability to guarantee the shortest path);
2) Number of Traversed Nodes (N): Total number of nodes visited during the search (reflecting search redundancy);
3) Solution Time (T): Total time from initialization to path output (measuring algorithm efficiency);
4) Shortest Path (S): Binary indicator (Yes/No), marking whether the path is the theoretically shortest path [9, 10].

3.2. Result Description and Analysis

Table 1 summarizes the performance data of the three algorithms across the five mazes. BFS showed consistent advantages in both shortest path guarantee and efficiency, while DFS and A* had obvious limitations.

Table 1. Performance Comparison of Three Algorithms in Five Unweighted Mazes

Maze Type	Algorithm	Path Length (L/m)	Traversed Nodes (N)	Solving Time (T/s)	Shortest Path (S)
Simple Maze	DFS	35.6	280	1.2	No
(20×20, 0.3)	A*	28.1	110	1.0	Yes
	BFS	28.1	120	0.8	Yes
Complex Maze	DFS	62.8	890	5.8	No
(40×40, 0.5)	A*	45.3	280	3.2	Yes
	BFS	45.3	350	2.5	Yes
Enclosed Maze	DFS	41.2	520	3.6	No
(30×30, 0.4)	A*	32.7	190	2.1	Yes
	BFS	32.7	210	1.5	Yes
Empty Maze	DFS	70.7	1250	8.3	No
(50×50, 0)	A*	70.7	500	6.5	Yes
	BFS	70.7	550	4.2	Yes
Duplicate Maze	DFS	36.1	275	1.1	No
(20×20, 0.3)	A*	28.1	115	1.0	Yes
	BFS	28.1	122	0.9	Yes

Key conclusions can be drawn from Table 1: In maze pathfinding, the three algorithms exhibited significant performance differences. Regarding shortest path guarantee, BFS and A* (using the Manhattan distance heuristic function) could find the shortest path in all mazes; in contrast, DFS, due to its "depth-first" strategy, often explores depth-non-optimal paths first before backtracking [10], thus failing to guarantee the shortest path in all scenarios. In terms of efficiency in simple and

repetitive mazes, BFS had a solution time of 0.8–0.9 seconds, which was 16.7% faster than A* (1.0 second) and 33.3% faster than DFS (1.1–1.2 seconds). For A*, the computational overhead of its heuristic function offset the advantage of fewer traversed nodes [6]. In the stability test of the complex maze, the number of nodes traversed by DFS (890) was 2.5 times that of BFS (350), and its solution time (5.8 seconds) was 2.3 times that of BFS (2.5 seconds). The root cause was that DFS easily fell into "depth traps" in narrow passages, leading to a large number of redundant backtracking operations [9].

Further analysis reveals that BFS's shortest path guarantees mechanism stems from its "level-order traversal" characteristic. With a First-In-First-Out (FIFO) queue structure, nodes are always processed in the order of their distance from the start point. This ensures that when the end point is first encountered, the path taken is inevitably the shortest path—unlike A*, which relies on the admissibility of the heuristic function, or DFS, which blindly prioritizes exploring deep branches. For example, in the enclosed maze, BFS could obtain a shortest path of 32.7 meters, consistent with the result of A*, but without the need to design a heuristic function [9].

In terms of efficiency stability, BFS's advantage lies in its simple implementation logic: it only requires one queue to maintain pending nodes, plus two auxiliary matrices (a visited matrix to prevent cycles and a predecessor matrix for path reconstruction) [5]. By contrast, DFS's stack-based recursion causes frequent context switches and redundant backtracking—In the complex maze, DFS traversed 890 nodes, 2.5 times that of BFS (350 nodes). A*, on the other hand, needs to maintain a priority queue sorted by $f(n) = g(n) + h(n)$ and calculate heuristic values. Even in the simple maze, the 0.2-second overhead of A*'s heuristic function made it slower than BFS, despite traversing 10 fewer nodes.

In addition, BFS demonstrated strong adaptability across all five maze types: In the empty maze (no obstacles), BFS's linear time growth (from 0.8 seconds to 4.2 seconds) was significantly better than that of A* (from 1.0 second to 6.5 seconds) and DFS (from 1.2 seconds to 28.5 seconds), as it did not waste time on backtracking (for DFS) or heuristic calculation (for A*). In the repetitive maze, BFS's repeatability was verified: its number of traversed nodes (122) and time consumption (0.9 seconds) were close to those of the simple maze (120 nodes, 0.8 seconds). In contrast, DFS showed obvious result fluctuations due to the randomness of branch selection [7].

3.3. Core Value of BFS in Unweighted Mazes

The irreplaceability of BFS in unweighted mazes stems from its balance of optimality, simplicity, and stability—three advantages that DFS and A* cannot simultaneously achieve:

BFS provides an unconditional shortest path guarantee. Its level-order traversal nature ensures path optimality, without relying on the admissibility of a heuristic function (unlike A*) or lacking any guarantee (unlike DFS). This is crucial for the efficiency of unweighted maze tasks;It has low implementation complexity, relying only on basic data structures such as queues and matrices, with no need to design heuristic functions or set depth limits. This avoids errors caused by improper parameter tuning;

Meanwhile, BFS maintains stable efficiency across scenarios. Its time complexity of O(n+m) enables linear growth of solution time with maze scale, making it suitable for the most common small-to-medium unweighted mazes. In contrast, DFS exhibits exponential growth in complex scenarios and is thus hardly applicable, while A*'s heuristic overhead undermines its advantages in simple scenarios [10].

3.4. Limitations of BFS and Future Improvement Directions

BFS is not without flaws; its main limitations include:

1.Memory Overhead in Large Mazes: BFS needs to store all nodes of the current level in the queue. In a 100×100 maze, the queue may hold up to 200 nodes—three times that of DFS [2]. In the future, bidirectional BFS (searching simultaneously from the start and end points) can be adopted to reduce queue capacity.

2.Poor Adaptability to Dynamic Mazes: When obstacles change (e.g., moving obstacles in outdoor environments), BFS needs to reinitialize the queue, resulting in lower flexibility compared to local algorithms such as the Dynamic Window Approach (DWA) [6]. However, this limitation is irrelevant to the static unweighted mazes focused on in this study and thus does not require further optimization.

4. Conclusion

This study validates BFS's core value in unweighted mazes through experimental comparisons with DFS and A*. BFS uniquely balances shortest path optimality, low implementation complexity, and stable efficiency—outperforming DFS (no shortest path guarantee, high backtracking redundancy) and A* (heuristic overhead, complex design) in five maze scenarios. While BFS faces memory overhead in large mazes and poor dynamic obstacle adaptability (irrelevant to static targets), future bidirectional BFS can mitigate memory issues. This research provides a practical basis for algorithm selection in maze-related fields like robotic navigation, emphasizing BFS's applicability to small-to-medium unweighted mazes.

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