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Conflict Analysis in Multi-source Uncertain Environments Based on Three-way Concept Analysis

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Abstract: Multiple sources use different attitude to the same agent and issue, which may introduce complex interpretation of agreement and conflict. This paper proposes a framework for complete, aligned evaluations on a common opposition-to-support scale. we use the minimum and maximum accepted evaluations form the source data. A definite attitude is assigned only when all source satisfies that the corresponding support or opposition threshold, and all other evaluations keep unresolved. The result construct a three-valued context and it is analyzed using conflict-oriented three-way concepts. The generated concept intents represent the agreement, confirmed conflict, and boundary issues, while the extents preserve these issue sets. The elementary properties of normalized proportions describe the effects of group enlargement, source addition, and nested threshold changes. Two case studies illustrate the role of the closure, and explain the distinction between reduced conflict and increased information. An arithmetic-mean comparison clarifies the different confirmation policies. The paper provides an interpretable description under three-valued formal context, it provides a solution for conflict analysis of multi-source information.

Keywords: Conflict Analysis; Multi-Source Information; Uncertain Attitudes; Three-Way Concept Analysis

1. Introduction

Conflict analysis examines how the attitudes of agents toward issues produce agreement and disagreement, and find a conflict resolution method to deal with the conflict. An issue may occur by many sources, such as represent a proposed action, an allocation decision, or a requirement under consideration. A group can agree in favor of an issue, agree against it, or contain agents with opposing positions. Effectively identifying and processing the conflict information is a research problem in conflict analysis. In 1998, Pawlak's rough-set treatment established a systematic approach to conflict information [1], and Yao subsequently reformulated and extended this perspective through three-way conflict analysis [2]. These studies motivate an explicit distinction between attitudes recorded for individual agents and relations inferred for a group.

The three-way decision methods propose a non-committal design in addition to acceptance and opposition, When the agent's attitude towards the issue is unclear, using the three-way decision methods can more effectively describe the information system. It allows an analyst to retain an intermediate state while making definite conclusions where the evidence permits them. However, the meaning of that state depends on how the evaluations are obtained and interpreted. A recorded neutral attitude, uncertainty about an attitude, and disagreement among observers are different situations. The connection between rough-set and formal-concept models developed by Lang et al. [3] provides a theoretical setting for relating evaluations to conflict categories. In the present paper, the intermediate agent label specifically denotes unresolved evidence after multiple accepted evaluations have been considered.

Formal concept analysis organizes objects through the attributes they share [4]. Its extent–intent representation offers a natural way to relate a group of agents to constraints on issues. Three-way formal concept analysis extends this description by incorporating additional negative or complementary information [5]. Concept-based conflict studies have examined group relationships under one-vote-veto settings [6] and clarified formal-concept perspectives on three-way conflict analysis [7]. Zhi et al. [8] introduced conflict-oriented three-way concepts for three-valued contexts and derived formulas connecting concept intents to group issue categories. We adopt that particular representation; its operators and characterization formulas are established tools rather than new results introduced here.

Uncertain and distributed evaluations have also received substantial attention. Zhi et al. [9] studied multi-level conflict analysis using fuzzy formal contexts. Qi et al. [10] investigated distributed conflict analysis with context fusion and varying analysis levels. Wang et al. [11] considered uncertain attitudes and third-party perspective with fuzzy concept lattices and conflict resolution methods. Tang et al. [12] developed an information-fusion model for multi-source fuzzy conflict data. These methods provide some methods for multi-source information processing and fuzzy information analysis, allowing us to analyze uncertain information from multiple sources.

Choosing the average is a very common choice, but when there are significant differences in attitudes between agents, the expression of the average becomes blurred. For example, the evaluations (0.95, 0.95, 0.25) have mean approximately 0.717 and range [0.25, 0.95]. With opposition and support thresholds of 0.3 and 0.7, respectively, a mean-based rule assigns support, while an all-source confirmation rule leaves the attitude unresolved. In this case, choosing to aggregate different decisions is not a good information processing method. Instead, using clustering or community division methods to divide different agents into different groups is beneficial for better identifying and processing this information.

In this paper, we design a method that assigns unambiguous attitudes only when all data meet relevant thresholds, while retaining the minimum and maximum acceptable assessments. In addition, we apply the conflict analysis-oriented operator in [8] to the method design. This method allows for a clear assessment of the differences in attitudes between different

agents in conflict environments. We give short proofs of their structural properties, distinguish a queried group from its generated extent, and state a direct computational procedure. Two examples show how the calculations can be reproduced and how source and threshold changes affect conclusions.

The contributions are as follows. We analyze the structure and nature of the proposed algorithm, and analyze the impact of changes in data and thresholds on conflict analysis in a case study. We clarified that an agent set with fewer conflicting outcomes does not necessarily indicate weaker conflicts in the agent set; smaller conflicts may reflect less decisive information. Therefore, our analysis of boundary information while identifying conflicts is consistent with the broader focus on different aspects of conflict assessment in [13].

The paper is organized as follows. Section 2 defines the data setting and the adopted concept representation. Section 3 specifies source envelopes, attitude assignment, issue characterization, properties, and computation. Section 4 is case studies, that presents two examples and threshold sensitivity. Section 5 conclude this paper and summarizes the interpretation supported by the analysis and its limitations. Table 1 shows the notation used in this paper.

Table 1 Notation used in this paper

Symbol	Meaning and convention
U, E, S; n, m, q	Agents, issues, accepted sources and their cardinalities; finite and nonempty
$r^{(k)}$, $M^{(k)}$	Source attitude evaluation in [0, 1] and aligned n × m matrix
$\underline{r}$, $\bar{r}$, Δ	Minimum, maximum and width across accepted sources
β, α; v, Kv	$0 \le \beta < \alpha \le 1$; labels −1, 0, +1 and three-valued context
I; G; $\mathcal{G}$, t	Binary incidence; nonempty query group; selected groups and their number
XG, YG, ZG	No opponent, no supporter, and all-definite common constraints
D; L	Common-description and inverse-derivation operators
cl(G), H, cG	Closure, generated extent and generated concept
A(G), C(G), B(G)	Agreement, confirmed-conflict and boundary issue sets
dA, dC, dB	Cardinalities of the issue sets divided by m
$\bar{r}^{mean}$, $\bar{v}$	Mean evaluation and comparison label in Case 2

2. Preliminaries

2.1. Multi-source attitude information

Let $U = \{u_1, \ldots, u_n\}$, $E = \{e_1, \ldots, e_m\}$, and $S = \{s_1, \ldots, sq\}$ be finite nonempty sets of agents, issues, and accepted sources, respectively. Here, sk has an attitude evaluation $r^{(k)}(u_i, e_j)$ in [0,1]. Its evaluation matrix $M^{(k)}$ has n rows and m columns, with entry $r^{(k)}(u_i, e_j)$. The value 0 indicates full opposition and 1 full support, and the intermediate values are graded evaluations on that same scale.

Every matrix has identical row and column meanings, and every source evaluates every agent–issue pair. We assume that these sources are fair third-party perspectives, rather than content influenced by existing data. We assume comparable scales and equal source standing for the confirmation rule. Neither reliability weights nor source frequencies determine whether an attitude is definite. Thresholds are specified before the reported analysis. No probability model, independence assumption, or ground-truth calibration is imposed. Related multi-source fuzzy settings occur in [10, 12], but the complete overlapping setting here is more specific than context merging through object or attribute extensions.

The notation table summarizes the objects used throughout. Agent, issue, source, and query-group indices are denoted by i, j, k, and h, respectively. Subscripts identifying a source set or a threshold pair are added only when comparing contexts. The symbols J, G′, F, and S_2 or S_3 are defined locally when needed.

2.2. Formal concept analysis and three-way concepts

A binary formal context KI = (U, E, I) consists of agents, issues, and an incidence relation $I \subseteq U \times E$. For $G \subseteq U$ and $F \subseteq E$, the classical derivations are [4]

$$G'_I = \{e \in E : \forall u \in G, (u,e) \in I\},$$
$$F'_I = \{u \in U : \forall e \in F, (u,e) \in I\}. \quad (1)$$

A formal concept (G, F) satisfies G′_I = F and F′_I = G. The extent G contains exactly the agents satisfying the intent F, and the intent contains all common attributes of the extent. These mutual conditions provide a closed description rather than an arbitrary group–attribute pairing. The binary context is reviewed only to establish the extent–intent viewpoint; it is not the context used for the calculations below.

Three-way concept analysis incorporates additional information into this viewpoint [5]. Several variants exist, so the choice of operators matters. We use the conflict-oriented three-component representation of [8], developed for a three-valued context. It differs from the original object-induced three-way concepts derived from binary incidence information. The connection between groups and bundles of issues discussed in [7] motivates its use here, while the precise adopted operators are defined next.

2.3. Conflict-oriented three-way concepts

Let Kv = (U, E, v) be a context with $v: U \times E \to \{-1, 0, +1\}$. For $G \subseteq U$, define the common issue constraints, following [8, Section 3], by

$$X_G = \{e \in E : \forall u \in G, v(u,e) \neq -1\},$$
$$Y_G = \{e \in E : \forall u \in G, v(u,e) \neq +1\},$$
$$Z_G = \{e \in E : \forall u \in G, v(u,e) \neq 0\},$$
$$D(G) = (X_G, Y_G, Z_G). \quad (2)$$

An issue belongs to XG if the group contains no definite opponent, to YG if it contains no definite supporter, and to ZG if every member has a definite label. Membership in XG alone does not establish unanimous support, similarly,

membership in YG alone does not establish unanimous opposition, there may be overlap between the data in both.

For any X, Y, Z ⊆ E, the inverse derivation is

$$L(X,Y,Z) = \{u \in U: \forall e \in X, v(u,e) \neq -1, \forall e \in Y, v(u,e) \neq +1, \forall e \in Z, v(u,e) \neq 0\}. \quad (3)$$

A conflict-oriented three-way concept (H, (X, Y, Z)) satisfies D(H) = (X, Y, Z) and L(X, Y, Z) = H. Its extent contains all agents that support the intent. For a selected nonempty group G, its generated extent and concept are

$$H = \mathrm{cl}(G) = L(D(G)),$$
$$c_G = (H, D(H)) = (\mathrm{cl}(G), D(G)). \quad (4)$$

The standard closure identities are

$$G \subseteq \mathrm{cl}(G),\ D(\mathrm{cl}(G)) = D(G),$$
$$\mathrm{cl}(\mathrm{cl}(G)) = \mathrm{cl}(G). \quad (5)$$

The concept starts with the smallest single object to calculate the properties supported by that object, and then calculates which objects support these properties together. Finally, we obtain the extension and connotation of a pair of mutually largest common sets, which are called conflict analysis-oriented three-way concepts. The operators used in the above calculations are the calculation methods of the conflict analysis-oriented three-way concepts [8].

Here, for a given threshold, in addition to the conflicting and non-conflicting sets, there are also neutral elements, which we call unresolved evidence.

2.4. Group-level agreement conflict and boundary issues

For a nonempty group G, an issue has label +1 or -1 for every member. A confirmed-conflict issue contains at least one definite supporter and opponent. We define

$$A(G) = \{e \in E : (\forall u \in G, v(u,e) = +1) \text{ or } (\forall u \in G, v(u,e) = -1)\},$$
$$C(G) = \{e \in E : \exists u, w \in G, v(u,e) = +1, v(w,e) = -1\},$$
$$B(G) = E \setminus (A(G) \cup C(G)). \quad (6)$$

Here, Agreement means that all members of the group agree, which also includes all members of the group jointly opposing a proposal, which demonstrates consistency within the group. confirmed refers to the truthfulness of the current problem from a third-party perspective, rather than the truthfulness of the problem in objective reality. The boundary set B(G) comprises the remaining issues, each of which has an unresolved label and no confirmed supporting–opposing pair.

An unresolved cell does not automatically make an entire issue a boundary issue: other members may already provide an opposing pair. Conversely, the absence of a confirmed pair does not establish full agreement. These distinctions retain the group-level interpretation of concept-based conflict categories [6, 8] while respecting the separation between attitude labels and relational categories emphasized in three-way conflict analysis [2].

3. Method

3.1. Source-envelope representation

The source envelope records the observed range for each agent–issue pair. Define

$$\underline{r}(u,e) = \min_{1 \le k \le q} r^{(k)}(u,e),$$
$$\overline{r}(u,e) = \max_{1 \le k \le q} r^{(k)}(u,e). \quad (7)$$

The corresponding interval and width are

$$J(u,e) = [\underline{r}(u,e), \overline{r}(u,e)],$$
$$\Delta(u,e) = \overline{r}(u,e) - \underline{r}(u,e). \quad (8)$$

All accepted evaluations lie in J(u, e). The envelope retains their extremes but not their frequency distribution or every interior value. Repeated identical evaluations do not alter it. This makes the representation suitable for an interpretation in which each accepted source supplies a constraint on confirmation. It is a modeling choice for the present workflow, motivated by uncertain and multi-source settings [9–12], rather than an attribution of this exact rule to those studies.

An envelope [0.2, 0.9] indicates a large spread across the supplied evaluations, whereas [0.55, 0.55] indicates identical intermediate evaluations. Both can be unresolved under the same thresholds, although the underlying reasons differ. The width Δ describes source spread only. It is neither a probability nor a statistical confidence width, and a zero width does not guarantee a definite attitude. In particular, the envelope is formed from finitely observed evaluations; it does not claim to bound every possible future evaluation or the agent's unobserved true attitude.

An extreme source has the same standing as every other accepted source. Consequently, a single low evaluation can block definite support and a single high evaluation can block definite opposition. This strategy makes the choice of a recognized third-party perspective important. Because conflict analysis methods themselves do not exclude edge information, but rather preserve and clearly present this information. Therefore, this strategy allows a third-party perspective to have a very clear understanding of the distribution of attitudes across the group and to make decisions based on this information.

3.2. Conservative three-valued attitude assignment

Choose opposition and support thresholds satisfying $0 \le \beta < \alpha \le 1$. The envelope produces the label

$$v(u,e) = +1 \text{ if } \underline{r}(u,e) \ge \alpha,$$
$$v(u,e) = -1 \text{ if } \overline{r}(u,e) \le \beta,$$
$$v(u,e) = 0 \text{ otherwise.} \quad (9)$$

Thus, +1 requires $r^{(k)}(u, e) \ge \alpha$ for every accepted source, while −1 requires $r^{(k)}(u, e) \le \beta$ for every accepted source. Here, there are only one condition can hold in one time. For example, it may require $\alpha \le$ the lower endpoint $\le$ the upper endpoint $\le \beta$, contradicting $\beta < \alpha$. For each fixed issue, the rule partitions agents into definite supporters, definite opponents, and unresolved agents, which is meet the general strategy of three-way decision [2, 3].

For $\beta = 0.3$ and $\alpha = 0.7$, evaluations (0.80, 0.85, 0.90) means that envelope [0.80, 0.90] and label +1, evaluations (0.10, 0.20, 0.25) mean [0.10, 0.25] and label −1, and the triple (0.95, 0.95, 0.25) yields [0.25, 0.95] and label 0. These examples reflect the ability of our method to retain marginal information. In the third case, low evaluations are not strictly discarded, but are retained in the final information.

Specifically, label 0 indicates that the multi-source information in this group cannot reach a consensus, that is, there is a conflict within the group. This also represents a characteristic of current conflict analysis methods, which retain a low proportion of marginal data, emphasizing the existence of conflicts in the current group.

3.3. Generated concepts and issue characterization

Apply the operator D and L to the formal context Kv. For each group, we compute its common constraints and generated extent before interpreting its issues. In the following, we applies the formulas of conflict analysis-oriented three-way concept to this context. We provide proof that this content is valid.

Theorem 1 Concept characterization of the three issue sets. Let G be a nonempty subset of U and H = cl(G). Then

$$A(G) = (X_G \cup Y_G) \cap Z_G,$$
$$C(G) = E \setminus (X_G \cup Y_G),$$
$$B(G) = (X_G \cup Y_G) \setminus Z_G. \quad (10)$$

Moreover,

$$A(H) = A(G),\ C(H) = C(G),$$
$$B(H) = B(G). \quad (11)$$

Proof. If e belongs to XG ∩ ZG, then all members have the label of 1 or 0, Hence all members have label +1. Conversely, consistent support guarantees that e appears in both sets. Similarly, the argument for YG ∩ ZG represent unanimous opposition. we can take the union of these alternatives and it gives the agreement formula.

Next, e lies outside XG precisely when some member is a definite opponent, and outside YG precisely when some member is a definite supporter. For the outside XG ∪ YG, there are two conflicting factors, which means that the set is conflicting. Regarding the border issue, since the agreement and conflict formulas exhaust ZG ∩ (XG ∪ YG) and E \ (XG ∪ YG), their remaining complement is (XG ∪ YG) \ ZG.

Lastly, D(H) = D(G) can be obtained by closure, so all three formulas give identical results for G and H.

This theorem provides a detailed definition of whether an intent partially overlaps or completely does not overlap, and in this way, the two can be distinguished. For example, for a question that cannot be opposed or supported, and that needs to be determined, if it does not appear in either direction, it indicates that there is a confirmation conflict. In this analysis, we did not construct a complete concept lattice because we did not need the partial order of the concept lattice to study the relationships between object sets. This can significantly save the time consumption of conflict analysis.

By using closure computation directly, we can obtain the maximum common properties that objects in any group have in a given formal context. Based on specific attributes, it can be determined whether the group is in a conflict, neutral, or allied state.

3.4. Normalized measures and basic properties

Normalize the issue counts by m = |E|:

$$d_A(G) = \frac{|A(G)|}{m},\ d_C(G) = \frac{|C(G)|}{m},$$
$$d_B(G) = \frac{|B(G)|}{m}. \quad (12)$$

Here, the sets A(G), C(G), and B(G) together constitute a complete E, where A(G)+C(G)+B(G)=1. During the calculation process, at least one support-opposition problem can always be found. Different subgroups within a group may be in different conflict states because different subgroups may contain different objects, and the maximum attributes calculated from these objects are not necessarily the same as those of the original group.

Assigning a fixed fraction of boundary issues to conflict would impose an additional interpretation of unresolved evidence. We therefore do not use the weighted inconsistency score of [8] as a default measure. The present triple distinguishes what is confirmed from what is unresolved without attaching a severity weight to either. Measures based on alliance and conflict functions [13] provide related motivation for retaining distinct aspects of a situation, but their specific functions are not implemented here.

Proposition 1 Group enlargement in a fixed context. For nonempty G ⊆ G′ ⊆ U,

$$A(G') \subseteq A(G),\ C(G) \subseteq C(G'). \quad (13)$$

Proof. A unanimous definite label on G′ remains unanimous when restricted to G, proving the first inclusion. A definite supporting–opposing witness pair in G remains present in G′, proving the second. Dividing cardinalities by the fixed number of issues gives dA(G′) ≤ dA(G) and dC(G′) ≥ dC(G). □

There is no general monotonic direction for B under group enlargement. An issue that is unanimous on a smaller group may become a boundary issue after adding an unresolved member. A boundary issue with labels (+1, 0) may instead become a conflict issue after adding a definite opponent. These possibilities concern the addition of agents within an unchanged context, and must be distinguished from the addition of evaluation sources.

Proposition 2 Addition of accepted sources. Suppose S ⊆ S′, the old evaluations are unchanged, and the thresholds are fixed. For every pair,

$$\underline{r}_{S'}(u,e) \le \underline{r}_S(u,e),$$
$$\overline{r}_{S'}(u,e) \ge \overline{r}_S(u,e). \quad (14)$$

For a fixed nonempty group G,

$$A_{S'}(G) \subseteq A_S(G),$$
$$C_{S'}(G) \subseteq C_S(G),$$
$$B_S(G) \subseteq B_{S'}(G). \quad (15)$$

Proof. Enlarging the set over which a minimum or maximum is taken gives Eq. (14). A previously definite label can remain definite or become unresolved. It cannot become the opposite definite label because the retained old evaluations already block that condition. A previously unresolved label also remains unresolved: adding constraints cannot establish a universal threshold condition that previously failed. Therefore, every definite label in the enlarged-source context already existed in the original context. New unanimity and new supporting–opposing pairs are impossible, proving the first two inclusions in Eq. (15). Taking complements of their union proves the boundary inclusion.

The proposition describes conservative confirmation, not a reduction of actual interpersonal conflict. The corresponding agreement and confirmed-conflict proportions cannot increase, while the boundary proportion cannot decrease. A conflict issue can disappear because a witness becomes unresolved even though no supplied earlier evaluation changes. Case 2 demonstrates why a decrease in dC must be interpreted together with the increase in dB.

Remark 1 When the threshold is more restricted, for example, $\beta' \le \beta < \alpha \le \alpha'$, then under the more restrictive threshold, only the confirmed labels can be retained as much as possible, and in extreme cases, they will all degenerate to 0. Conversely, a more lenient threshold will reduce the proportion of labels with 0, making it easier to determine whether a group is in a state of conflict. However, an overly lenient threshold will lead to significant conflicts in groups with weaker conflicts, which may increase the difficulty of conflict analysis and conflict resolution from a third-party

perspective. Therefore, the choice of threshold affects the judgment of the group's nature, and thresholds that are too strong or too weak will reduce the retention of valid information.

Remark 2 If q=1, or if all agents from all sources have the same attitude towards events, then all closures cannot obtain valid information, and the actual result will degenerate into a single point. Then the entire conflict analysis method will degenerate into single-source information processing. This common single-source information processing has been addressed in many existing conflict analysis studies based on formal concept analysis. Current algorithms are backward compatible with single-source information processing when there is only one source.

3.5. Computational procedure

In table 2, algorithm 1 provide the algorithm for compute the conflict analysis results from multi-source information. The group list $\mathcal{G}$ = {G₁, …, Gt} is used to analysis. where all groups are nonempty. Here, the algorithm can reflect the attitudes of all agents from all sources towards events. Finally, the algorithm returns the classification results of the conflict analysis, representing the conflict states of all groups.

Table 2 Algorithm for Conflict analysis

Algorithm 1 Conflict analysis of selected groups	
Input: $M^{(1)}$,…, $M^{(q)}$; β < α; nonempty groups $\mathcal{G}$.	
Output: cG, A(G), C(G), B(G), and proportions.	
1	For each pair, compute source min and max.
2	Assign v by Eq. (9).
3	For each selected G in $\mathcal{G}$:
4	Compute (XG, YG, ZG) = D(G).
5	Compute H = L (XG, YG, ZG).
6	Record cG = (H, (XG, YG, ZG)).
7	Recover A, C, B using Eq. (10).
8	Compute dA, dC, dB using Eq. (12).
9	Return results.

The total time complex of this algorithm is O(kmn). For a group G, computing D(G) takes O(|G|m). A direct scan of all agents computes L(D(G)) in O(mn), while recovering issue sets and proportions costs O(m). For the given t groups, the total cost is

$$O(qmn + \sum_{G\in\mathcal{G}}|G|m + tnm) = O(qmn + tnm). \quad (16)$$

In addition, some auxiliary tasks also accumulate event complexity, but not more than the maximum time complexity. For example, the auxiliary storage effort consumes O(mn+n+m), where the label context uses O(mn) and the study problem set consumes O(n+m). Although these temporal complexities arise during data processing, not during the computational process of conflict analysis. However, data processing is a necessary step in data analysis, so these event complexities also need to be categorized into the overall temporal complexity.

4. Case studies

4.1. Group analysis in a planning committee

We present a case 1 to demonstrate our method. Here, we consider the attitudes of the six committee members towards five issues. Of these, six members are: Economic Member (u1), Environmental Member (u2), Healthcare Member (u3), Construction Member (u4), Management Member (u5), and Implementation Member (u6). They will discuss five topics: facility expansion (e_1), additional funding (e_2), accelerated timetable (e_3), outsourcing (e_4), and additional monitoring requirements (e_5). Table 3 shows the attitudes of the three assessment channels towards these issues.

Set β = 0.3 and α = 0.7. For (u_1, e_1), the attribute [0.80, 0.90] stand for +1. For (u_1, e_3), [0.10, 0.20] becomes −1. For (u_1, e_4), [0.10,0.90] yields 0 because neither universal confirmation condition holds.

The lower endpoint for (u_3, e_5) is α = 0.70, so the included-threshold convention gives +1. For (u_5, e_4), all sources give 0.50. The results here show that when all results are the same, there is no conflict within the group. The algorithm in this paper degenerates into single-source information processing. Depending on the threshold, the algorithm in this paper will classify it as a neutral or allied group, rather than classifying it as a conflict.

Table 3 Synthetic source evaluations for Case 1

Agent	e_1	e_2	e_3	e_4	e_5
	0.90	0.80	0.10	0.90	0.80
u1	0.85	0.75	0.15	0.50	0.90
	0.80	0.85	0.20	0.10	0.85
	0.80	0.75	0.20	0.45	0.85
u2	0.90	0.80	0.25	0.50	0.80
	0.85	0.90	0.10	0.55	0.90
	0.75	0.10	0.15	0.40	0.70
u3	0.80	0.20	0.20	0.50	0.80
	0.85	0.25	0.10	0.60	0.75
	0.10	0.85	0.80	0.20	0.10
u4	0.20	0.80	0.85	0.50	0.15
	0.15	0.75	0.90	0.80	0.20
	0.20	0.75	0.85	0.50	0.25
u5	0.25	0.85	0.80	0.50	0.20
	0.15	0.80	0.75	0.50	0.10
	0.40	0.60	0.20	0.50	0.20
u6	0.50	0.65	0.30	0.50	0.50
	0.60	0.70	0.40	0.50	0.80

The three valued formal context at (β, α) = (0.3, 0.7) is shown in Table 4.

Table 4 Three-valued context at (β, α) = (0.3, 0.7)

Agent	e_1	e_2	e_3	e_4	e_5
u1	+1	+1	−1	0	+1
u2	+1	+1	−1	0	+1
u3	+1	−1	−1	0	+1
u4	−1	+1	+1	0	−1
u5	−1	+1	+1	0	−1
u6	0	0	0	0	0

The six query groups are

$$G_1 = \{u_1, u_2\},\ G_2 = \{u_1, u_2, u_3\},$$
$$G_3 = \{u_4, u_5\},\ G_4 = \{u_1, u_4\},$$
$$G_5 = \{u_1, u_6\},\ G_6 = U. \quad (17)$$

Consider G₄ in detail. Its description is

$$X_{G_4} = \{e_2, e_4\},\ Y_{G_4} = \{e_4\},$$
$$Z_{G_4} = \{e_1, e_2, e_3, e_5\}. \quad (18)$$

This requires the label on e4 to be 0, and since e2's attitude cannot be clearly opposed, therefore, the Z constraint requires definite labels on e_1, e_2, e_3, and e_5. Agents u_1, u_2, u_4, and u_5 satisfy all of these requirements. Agent u_3 not satisfies the condition on e_2, and u_6 also not satisfies the definiteness

requirements. Thus,

$$\mathrm{cl}(G_4) = \{u_1, u_2, u_4, u_5\}. \quad (19)$$

Together with Eq. (18), this extent defines cG_4. By Theorem 1,

$$A(G_4) = \{e_2\},\ C(G_4) = \{e_1, e_3, e_5\},\ B(G_4) = \{e_4\}. \quad (20)$$

The proportions are (0.2, 0.6, 0.2). This result shows that for the four agents u1, u2, u4, and u5, they can basically form a stable alliance because their attitudes towards e4 are unclear only, but their attitudes towards other events are clear. From a third-party perspective, if we can help these four agents reach a consensus on e4, then these four agents can reach a stable alliance.

Table 5 Generated extents and group issue results

Group	H	Issue sets	dA; dC; dB
G1	1, 2	A: 1, 2, 3, 5 C: ∅ B: 4	0.8 0.0 0.2
G2	1, 2, 3	A: 1, 3, 5 C: 2 B: 4	0.6 0.2 0.2
G3	4, 5	A: 1, 2, 3, 5 C: ∅ B: 4	0.8 0.0 0.2
G4	1, 2, 4, 5	A: 2 C: 1, 3, 5 B: 4	0.2 0.6 0.2
G5	1, 2, 6	A: ∅ C: ∅ B: 1, 2, 3, 4, 5	0.0 0.0 1.0
G6	1, 2, 3, 4, 5, 6	A: ∅ C: 1, 2, 3, 5 B: 4	0.0 0.8 0.2

The generated extents and group issue results are shown in Table 5. Here, both G_1 and G_3 have a consistent ratio of 0.8, although their consistent positions on e_1, e_3, and e_5 are opposite. This confirms that our algorithm focuses on studying whether agents share the same attitude, rather than whether they jointly support a proposal. $\{u_1, u_2, u_6\}$, but there is no confirmed conflict or agreement, which means that the attitudes among these three agents are at a reconcilable level, and all five of their issues are boundary issues. At the same time, this also illustrates that the absence of conflict does not necessarily mean that an agent set is allied; they may also all be neutral.

As can be seen from Table 4, u_6 maintains a neutral stance on all issues, but the entire G_6 group still has conflicts on four issues. This is because the attitudes of other agents can influence the conflict analysis results of their group, which may cause completely neutral u_6 to exhibit conflict with others in some groups. For example, for an already conflicting group, joining u_6 does not necessarily transform the group into a neutral or allied state.

4.2. Source disagreement and conservative confirmation

In this case study, we provide a separate context with U = $\{u_1, u_2\}$ and E = $\{e_1, e_2\}$. Similar to above, we set $\beta = 0.3$ and $\alpha = 0.7$, and analyze G = U. Let accept $S_2 = \{s_1, s_2\}$, then add s_3 to obtain S_3, this represents expanding an existing information system into a multi-source information system. These data are shown in Table 6.

Table 6 Independent data for Case 2

Pair	s_1	s_2	s_3
(u_1, e_1)	0.95	0.95	0.25
(u_2, e_1)	0.10	0.20	0.10
(u_1, e_2)	0.90	0.90	0.90
(u_2, e_2)	0.95	0.95	0.25

Under S_2, the label rows for u_1 and u_2 are (+1, +1) and (−1, +1), respectively. Then, the group agrees on e_2 and has conflict on e_1.

After we adding s_3, it can change two envelopes to [0.25,0.95]. Combining all the multi-source information, the results in each row become (0, +1) and (−1,0). Here, both problems are boundary problems, so the results change when new information is added.

To clarify the aggregation policy, define an arithmetic-mean comparison within this case:

$$\bar{r}^{\mathrm{mean}}(u,e) = \frac{1}{q}\sum_{k=1}^{q} r^{(k)}(u,e),$$
$$\bar{v}(u,e) = +1 \text{ if } \bar{r}^{\mathrm{mean}}(u,e) \geq \alpha,$$
$$\bar{v}(u,e) = -1 \text{ if } \bar{r}^{\mathrm{mean}}(u,e) \leq \beta,$$
$$\bar{v}(u,e) = 0 \text{ otherwise.} \quad (21)$$

Here $\bar{r}^{\mathrm{mean}}$ denotes the mean of the data, the superscript distinguishes the two quantities. Throughout the formal context, all definitions and conceptual formulas are applied to the same mean to ensure that all calculations are consistent.

This result represents the average of the information in the multi-source information. Although the average will smooth out the boundary information, we showed in the previous case analysis that our algorithm can preserve the boundary information. Therefore, using the average here can illustrate the degree of uniformity of all the information from another perspective.

For each triple (0.95, 0.95, 0.25), the mean is 2.15/3 ≈ 0.7167, which exceeds α. The other two means are 0.1333 for (u_2, e_1) and 0.9000 for (u_1, e_2). Hence the mean-labeled rows under S_3 are back to (+1, +1) and (−1, +1). Table 7 compares the resulting group classifications.

Table 7 Rule and source comparison for G = U

Rule and sources	A; C; B	dA; dC; dB
Envelope S_2	{e_2}; {e_1}; ∅	0.5; 0.5; 0.0
Envelope S_3	∅; ∅; {e_1,e_2}	0.0; 0.0; 1.0
Mean S_3	{e_2}; {e_1}; ∅	0.5; 0.5; 0.0

This computational method allows us to process conflicting information from multiple perspectives. The algorithm presented in the previous section preserves the edge information, ensuring that all information is not lost. In this summary, the average is used to process all information. This result reflects the group's average attitude, which can improve the accuracy of information while reducing the loss of edge information.

The combination of the two methods can better model multi-source information and effectively deal with the conflict analysis problem of multi-source information. The relevant results analyze the problem from two perspectives, which facilitates a third-party perspective to grasp the problem from a more comprehensive perspective and thus make appropriate judgments.

4.3. Threshold sensitivity and discussion

Continue with Case 1, we here compare three threshold

pairs: the restrictive setting (β, α) = (0.2, 0.8), the main setting (0.3, 0.7), and the relaxed setting (0.4, 0.6). Table 8 lists the G_2, G_4, G_5, chosen to show different responses.

Table 8 Nested threshold analysis in Case 1

(β, α)	Group	dA	dC	dB
(0.2, 0.8)	G2	0.0	0.0	1.0
(0.3, 0.7)	G2	0.6	0.2	0.2
(0.4, 0.6)	G2	0.6	0.2	0.2
(0.2, 0.8)	G4	0.0	0.6	0.4
(0.3, 0.7)	G4	0.2	0.6	0.2
(0.4, 0.6)	G4	0.2	0.6	0.2
(0.2, 0.8)	G5	0.0	0.0	1.0
(0.3, 0.7)	G5	0.0	0.0	1.0
(0.4, 0.6)	G5	0.4	0.0	0.6

For G_2, an overly strict threshold invalidates the conflict analysis results, making it impossible to obtain valid information. Under the other two threshold designs, consistency is maintained on three issues, while conflict occurs on one issue. For G_4, there are three conflicting problems under all three thresholds. In particular, under stricter thresholds, the confederation state on e_2 cannot be maintained. At the same time, the boundary ratio increases from 0.2 to 0.4.

These examples illustrate that the choice of threshold can affect the results of conflict analysis; overly lenient or overly strict thresholds can reduce the content of valid information in conflict information.

Specifically, stricter thresholds retain less information about conflicts or alliances, which means relatively fewer definitive conclusions. Conversely, if a valid conflict that can be handled cannot be obtained under a strict threshold, then a more lenient threshold needs to be selected to obtain more information. In [9], Zhi also proposed the concept similar to decision cost. Under a given cost, an appropriate threshold is selected to conduct conflict analysis under the maximum conflict information, and an appropriate conflict resolution strategy is obtained. If the decision-maker only needs to process the least conflicting information at a lower cost, a more stringent threshold can be chosen. If decision-makers still have sufficient costs, they can choose a more lenient threshold and obtain more information.

5. Conclusion

This paper proposes a conflict analysis strategy for multi-source information and designs a conflict analysis method that preserves edge information. Case studies show that the current algorithm can effectively detect conflicts in multi-source information.

Specifically, first, by preserving the minimum and maximum attitudes among multi-source information, it is ensured that all information remains in the last conflicting information. Secondly, distinguish between support, opposition, and uncertainty by setting thresholds. Finally, the use of conflict analysis to guide three concepts provides the maximum public attitude description for each group.

The case study shows that the algorithm can handle conflict information in multi-source information and construct a unified conflict analysis result. Two case studies present results with edge information and results using mean framework, respectively. Together, these two results provide third-party decision-makers with comprehensive conflict information. In addition, the third case study illustrates that using different thresholds can affect the amount of valid information in the conflict analysis results.

In summary, this paper provides an idea for multi-source information conflict analysis and designs an effective strategy for processing edge conflict information. Future studies will investigate the selection of the optimal threshold given multivariate information.

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