Strategic Analysis of Dollar Auction

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Abstract. Shubik invented the dollar auction in 1971, which is a straightforward but practical auction model for elucidating the motivations and process of conflict escalation. Later on, several researchers used this paradigm to conduct the study on a variety of subjects. Waniek, for example, investigated the relationship between spitefulness and dollar auction in-depth and presented the optimal decision under different scenarios. In this paper, I simplified his results and merged them into simpler forms. Some proposals that may be advantageous to our economic environment are provided based on the performance.

Keywords: Dollar Auction, Game Theory, Operations Research.

1. Background

1.1 The Introduction of the Problem

Dollar auction, which was designed by economist Martin Shubik, illustrates a paradox of rational choice theory. As early as the end of the 20th century, some scientists carried out psychological experiments using the dollar auction, and the results showed that the players tend to be irrational during the auction. Nevertheless, many events, in reality, are similar to dollar auctions such as the international escalation. People in this situation may think, "Our dead shall not have died in vain." As people at the all-paid auction like the dollar auction thought, our money can't be spent for nothing. In this stage, compared with maximizing their own profit, players prefer to minimize the opponent's profit to “avenge”. A new index called the spite coefficient developed by Levine (1997) could be introduced to quantify people’s spitefulness.

For different degrees of spitefulness, what strategies should players use in the dollar auction? Previous literature has studied a lot among these and found that spitefulness definitely has a significant impact on the process and results of the dollar auction. All the studies reviewed so far, however, suffer from the fact that their research on the best strategy has not been simplified and made into a one-to-one form. Players could not easily execute the strategies and obtain the corresponding results with the complete information setting.

In my review, I will summarize the existing research on dollar auction and spitefulness, and present the strategy in a clearer way like the formula and table. At the same time, I will give different auction results under different situations according to the strategy, so as to help players better execute the strategy and obtain the desired results.

1.2 Literature Review

Previous work on game theory before Shubik (1971) has been limited to the gut-driven, intuitive field. He first developed the dollar auction and affected many areas of defense analysis. The logic of escalation in auctions clearly applied to a range of defense strategy problems due to his distribution. Teger (1980) is a pioneer in providing an experiment using dollar auction as the paradigm for a collection of controlled experiments on the construct of gradual commitment because of past investment. In his experiment, the condition of the participants was investigated and the psychological reasons for his or her irrational strategies were first obtained. O'neill (1986) expressed the strategies in dollar auction in a more technical way and derived a formula for the players' rational strategies and pointed out the very fact that rational players wouldn't bid against one another. He challenged the widely held view that the dollar auction is itself a trap, causing conflict by its design, arguing that the

dollar auction has an immediate solution in pure strategies which means that it should not touch off conflict escalation theoretically, however, exploits some irrationality within the bidders' behavior. Additionally, he compared the dollar auction to international political and military events, likening it to the conflicting state like the international escalation, in which invest resources would not be repaid. Carrying on the research about the international escalation and the dollar auction, Kirshina (1997) focused his research on the level of war of attrition. For this goal, he discovered sufficient conditions for the existence of symmetric monotonic equilibrium bidding strategies and tested the overall performance of all-paid auctions in terms of the predicted revenue accruing to the seller. Levine (1998) examined the results of ultimatum experiments and the final round of a centipede experiment and they become the first who succeeded in stimulating comparatively accurately what the distribution of altruism and spite within the population is, and quantifying people's behavior in the market. Later, Levine (2004) covered a lot of the main ideas and highlights about auctions in his paper, using a relatively game-theoretic approach. Brandt (2005) creatively attempted to study the bidding behavior of spiteful agents. This is a superb analysis for competitive scenarios similar to in closed markets wherever the loss of a competitor can possibly lead to future gains for oneself. They derived symmetric Bayes Nash equilibria for spiteful agents in 1st-price and 2nd-price sealed-bid auctions. Based on these equilibria they studied, the conclusion that the bidders' valuations reduced revenue in 2nd-price auctions, whereas it's the alternative impact in 1st-price auctions was drawn out. Beginning in 2015, Waniek did several contributing works about dollar auction. In 2015, Waniek questioned whether the step-up in the dollar auction is evoked by meanness and confirmed this conjecture in a variety of scenarios. Besides, in 2016, Waniek investigated the repeated model of dollar auctions, wherein the kind of the opponent and their degree of rationality isn't always recognized earlier. And they proved that when a non-spiteful participant bids in opposition to a malicious one, the game converges to a Nash equilibrium if each gamer used the strategy they invented to place their bids. In 2017, Waniek investigated the dollar auction model with spiteful players for the complete information setting, both for the standard situation and for the scenario in which the auction starts with non-zero bids. Their consequences deliver them insight into the possible outcomes of meanness onto conflict escalation. In addition, Migheli (2012) proposed the concept of one-dollar auction game excluding escalation. His findings suggest manifestations of some risk aversion, which is triggered by an English auction escalation, but when escalation is kicked out, expectations and indifference to preferences can lead to the same. This paper studies the dollar auction from another dimension instead of escalation as we usually see.

2. Preliminaries

2.1 Sketch of Auctions

Auction is held sale in public at which items are sold to the highest bidder. It's also regarded as one of the most successful applications of game theory, particularly in the context of games with incomplete information. There are several kinds of auctions, the most common of these are English Auction, Dutch Auction, First-Price Sealed Bid Auction, Second-Price Sealed Bid Auction, and so on.

2.1.1 English Auction

English auction is a selling method in which an asset is sold by inviting bids of increasing value. Before the auction begins, a base price or minimum is set and bidders must start bidding above this base price. The object is sold to the highest bidder. Unlike sealed auctions, English auctions are completely open and transparent. This means that all bidders are informed of one other's identities. During the auction, the value of each bid and the bidder who placed it is known to every bidder. Usually, each bidding will have a specified minimum increment, and the bidder can only increase the price by this increment multiplier. The surplus for the bidders can be presented as follow:

$$s_i(v_i) = \begin{cases} v_i - b_i & if\ b_i > \max\limits_{j \neq i} b_j \\ 0 & otherwise \end{cases} \quad (1)$$

Therefore, the optimal strategy for the bidders should be once the bidding price reaches the budget, leave immediately. So, the general result is that the person with the highest budget wins the auction and pays the budget price of the person with the second-highest budget.

2.1.2 Dutch Auction

The Dutch auction is often called Descending Price auction. During the process, the auctioneer starts with the highest price then the bids are received for unceasingly cheaper prices till all things are sold-out. This kind of auction is usually used for bulk sale items, in order that the whole lot is able to be sold at once. The surplus for the bidders can be represented as follow:

$$s_i(v_i) = \begin{cases} v_i - b_i & if\ b_i > \max\limits_{j \neq i} b_j \\ 0 & otherwise \end{cases} \quad (2)$$

$$E[s_i(v_i)|b_i] = (v_i - b_i) \times Pr(b_i) \quad (3)$$

As we can see above, if the bidder raises his hand once the price heats his valuation, then he can't get any profit in this auction. So, it is wiser for the bidder to shade their bid. In general, we are unable to predict the transaction price only depending on the budget of the bidders but we can guess it would be a little lower than the highest budget of the bidders.

2.1.3 First-Price Sealed Bid Auction

In this type of auction, the bidders submit the bids in a sealed envelope, and when the envelope unfolds the sale is created to the highest bidder who would pay the price he wrote. It's additionally called Blind Auction as a result of any bidder doesn't grasp the bid submitted by different bidders. The surplus for the bidders can be shown as follow:

$$s_i(v_i) = \begin{cases} v_i - b_i & if\ b_i > \max\limits_{j \neq i} b_j \\ 0 & otherwise \end{cases} \quad (4)$$

$$E[s_i(v_i)|b_i] = (v_i - b_i) \times Pr(b_i) \quad (5)$$

Similar to the Dutch auction, it would be more beneficial if the bidders shade their bid. Generally speaking, if there are more bidders or the other bidders are richer, shading less would be a better choice.

2.1.4 Second-Price Sealed Bid Auction

Second-Price Sealed Bid auction, which is also known as the Vickrey Auction, is exactly just like the First-Price Sealed Bid auction. The only distinction is that during this auction the highest bidder will pay the amount of the second-highest bid instead of his own one. The surplus for the bidders can be presented as follow:

$$s_i(v_i) = \begin{cases} v_i - \max\limits_{j \neq i} b_j & if\ b_i > \max\limits_{j \neq i} b_j \\ 0 & otherwise \end{cases} \quad (6)$$

$$E[s_i(v_i)|b_i] = (v_i - b_i) \times Pr(b_i) \quad (7)$$

A Second-Price Sealed Bid Auction is used in many existing blockchain domain name exchanges. Sealed voting and the second price mechanism solve the problem of "price sniping", and then encourage competitors to put forward their "perceived value of the bid" and "really willing to pay". In this way, participants will not be influenced by each other to continuously raise the bid price or falsely claim the price they are actually willing to pay.

2.1.5 All-Pay Auction

The mechanism of this auction is the bidder submits the bid b_i, and the highest bidder wins the bid. However, bidders must pay their bids whether they win the auction or not. Although these rules may seem odd, it is very useful when used as a model for lobbying or explaining political influence. The surplus can be shown as follow:

$$s_i(v_i) = \begin{cases} v_i - \max\limits_{j \neq i} b_j & if\ b_i > \max\limits_{j \neq i} b_j \\ -b_i & if\ b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{8}$$

The one-dollar auction is a kind of All-Pay auction, which uses a fixed stake of one dollar. That is, v=1 for all i. The surplus can be shown as follow:

$$s_i(v_i) = \begin{cases} 1 - \max\limits_{j \neq i} b_j & if\ b_i > \max\limits_{j \neq i} b_j \\ -b_i & if\ b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{9}$$

2.2 Spitefulness

Levine (1997) established an altruism theory in which players' payoffs are linear in both their own and their opponent's monetary income. He used the utility function to represent the people's altruism performance:

Considering n person participating in the games with players $i = 1, \ldots, n$. At the end of the game, player i receives a *direct utility* of u_i. Besides, player i also has a coefficient of altruism a which formed an adjusted utility of

$$U_i = u_i + \sum_{j \neq i} \frac{a_i + \mu a_j}{1 + \mu} u_j \tag{10}$$

where $0 \leq \mu \leq 1$. The aim for the bidders is to maximize U_i. The adjusted utility not only reflects the player's own utility, but also their regard for the other players. If $a_i > 0$ the player is considered to be altruistic, since such a player has positive regard for his opponents. If $a_i = 0$, the player is considered to be selfish, cares nothing related to his opponents as the usual case in assumptions. If $a_i < 0$ the player is supposed to be spiteful. The coefficient $\mu \in [0,1]$, reflects that the players are not able to have a higher regard for their opponents than for themselves. This ensures the same psychological state as people really are, it's another kind of selfishness.

Making Use of the results of ultimatum tests and the last round of a centipede experiment, Levine can pin down the distribution of altruism and spite in the population.

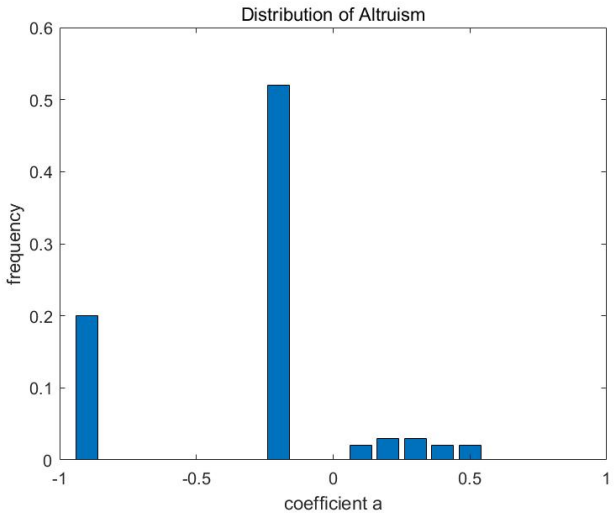


Figure 1. Distribution of Altruism

Levine obtained the distribution figure above, which surprisingly showing that a large number of individuals have relatively negative coefficients a_i, which means they are spiteful. Moreover, 20% of the population with a mean coefficient of -0.9, meaning that their spitefulness level is quite high. This result contradicts to our assumptions that "the players are selfish and only care about themselves" in Nash equilibrium before. Since the players are indifferent, they can transfer money to or from their opponent at no cost at the Nash equilibrium. The deviation from Nash equilibrium depends on the spitefulness or altruism of the marginal player. When a spiteful player is a marginal player, he will tend to move money away from his opponent. The opponent's plan will have to be adjusted to prevent him from doing so. As a result, in a symmetric game, this yields that equilibrium payoffs will be higher. When the altruist player is a marginal player, he will not do so. In the equilibrium case with purely selfish players, payoffs will be lower. Among the players, most of them tend to be spiteful instead of altruistic. Therefore, researching the spitefulness of the players is of high significance.

Following the work of Waniek (2017), we assume that a spiteful agent is not only interested in maximizing his own profit but also decreasing the profit of his opponent. More formally, the utility of player i can be represented as:

$$ui = (1 - a_i)p_i - a_i p_j \tag{11}$$

Where p_i and p_j are the profits of the respective players, and $a_i \in [0,1]$ is the spite coefficient of player i. This coefficient indicates the importance of the opponent's losing to player i. When $a_i = 0$, player i simply maximizes his own profit, in which case i is said to be a selfish (non-spiteful) player. When $0 < a_i \leq 1$, player i is said to be a spiteful player. Additionally, when $a_i = 1$, player i is considered to be a malicious player. Because he is solely interested in minimizing the profit of the opponent.

More formally, the profit of the winning player i can be defined as the difference between the bidding price and the valuation of one dollar. Assuming there are only two players in the auction, the winner among them can receive the value v, $b_i > b_j$, then $p_i = d - b_i$, $p_j = -b_j$. In summary, we can write the utility of the player i like this:

$$u_i = \begin{cases} a_i b_j + (1 - a_i)(v - b_i) if\, b_i > b_j \\ a_i(b_j - v) + (1 - a_i) b_i if\, b_i < b_j \end{cases} \tag{12}$$

3. Dollar Auction with Non-Spiteful Players

O'Neill (1986) pointed out that in an auction with limited budget, the starting player has a rational bidding strategy that ensures him to win. Assuming that both the players have the same budget B, then the strategy can be shown that the first player should make a bid of

$$(B - 1) mod (v - 1) + 1 \tag{13}$$

The second player should pass, as he will never be able to win the dollar in any situation. This is a study based on the situation that both players have a spite coefficient of 0. In other words, both players are "selfish", they only want to maximize their own profits and don't care whether the opponent's profits are large or small.

Based on O'Neill's strategy, Waniek (2015) makes an in-depth exploration of the optimal strategy for people with different degrees of spitefulness to face their selfish opponents.

3.1 Malicious Player $(a = 1)$ vs. Non-Spiteful Players $(a = 0)$

When a malicious player j $(a = 1)$ plays against a non-spiteful player i $(a = 0)$who follows the rational strategy we mentioned above, the utility function of the malicious player is

$$u_j = -p_i \tag{14}$$

The goal of player j is to maximize the loss of player i, no matter what the cost. Waniek (2015) demonstrated that the malicious player j's best tactic is to entice the non-spiteful player to continue bidding for as long as possible. In the end, it is the malicious bidder that gets the dollar. I think what needs to be emphasized here is, that if the malicious player is the first player, he is unable to achieve the outcome of the non-spiteful player's failure to win non-spiteful player. Therefore, given the chance to move first, the malicious player will always pass the move to the opponent. In this way, the malicious one can always be the second player. The optimal strategy for the malicious player to play against the non-spiteful player can be shown as follow.

$$b_j = \begin{cases} b_i + 1 \ \ if\ b_i < B - (v-1) \\ B \quad otherwise \end{cases} \tag{15}$$

3.2 Strongly-Spiteful Player $(\frac{1}{2} < a < 1)$ vs. Non-Spiteful Players $(a = 0)$

A player j with spite coefficient smaller than 1 but bigger than $\frac{1}{2}$ is called the strongly-spiteful player. When competing against a non-spiteful opponent using O'Neill (1986)'s rational strategy, a strongly-spiteful player can act like a non-spiteful player if he starts the auction for extremely specified budget and dollar values, according to Waniek (2015)'s argument. However, in most cases, the strongly-spiteful player is a malicious player with a spite coefficient of 1, and he keeps tempting the non-spiteful player to raise his bids, ensuring that the strongly-spiteful player wins the game. The optimal strategy for the strongly-spiteful player j depends on the order of the player.

$$if\ the\ non-spiteful\ player\ i\ bids\ first:\ \ follow\ the\ strategy\ for\ the\ malicious\ player$$

$$if\ the\ strongly-spiteful\ player\ j\ bids\ first:$$

$$b_j = \begin{cases} b_i \qquad if b_i < \frac{a_j}{1-a_j}(v-1) - \frac{2a_j - 1}{1-a_j}B \\ follow\ the\ strategy\ for\ the\ malicious\ player(15) \quad otherwise \end{cases} \tag{16}$$

Where t b_i is the optimal initial bid of a non-spiteful player following (13).

3.3 Moderately-Spiteful Player $(a = \frac{1}{2})$ vs. Non-Spiteful Players $(a = 0)$

A player j with spite coefficient equal to $\frac{1}{2}$ is called the moderately-spiteful player. When bidding with the non-spiteful player who follows the rational strategy derived by O'Neill (1986), the optimal strategy for the moderately-spiteful player j is proved by Waniek (2015).

$$if\ the\ non-spiteful\ player\ i\ bids\ first:$$

$$b_j = b_i + 1\ and\ finally\ raise\ his\ bid\ by\ v-1$$

$$if\ the\ moderately-spiteful\ player\ j\ bids\ first:$$

$$follow\ the\ strategy\ for\ the\ non-spiteful\ player$$

3.4 Weakly-Spiteful Player $(0 < a < \frac{1}{2})$ vs. Non-Spiteful Players $(a = 0)$

From the figure above, we can obtain the fact that most part of our population are weakly-spiteful players whose spite coefficient is between 0 and $\frac{1}{2}$. When they play against the non-spiteful player i, the optimal strategy is proved by Waniek (2015).

$$if\ the\ non-spiteful\ player\ i\ bids\ first:$$

$$b_j = \begin{cases} b_i + v - 1 \ \ if\ b_i < \frac{a_j B}{1-a_j} + 1 \\ pass \qquad otherwise \end{cases} \tag{17}$$

$$if\ the\ weakly-spiteful\ player\ j\ bids\ first:$$
$$follow\ the\ strategy\ for\ the\ non-spiteful\ player \quad (13)$$

4. Dollar Auction in Alternative Settings

Waniek (2017) simplified the strategies setting when it comes to alternative settings. The number of major strategies was reduced to four, which are denoted as Selfish Strategy, Malicious Strategy, Overbidding Strategy, and Pass respectively.

4.1 The Selfish Strategy

The Selfish Strategy is invented for the players with zero spitefulness. This means that these players do not care about the benefits of others, but only want to maximize their own benefits, they are called the non-spiteful player in our review above. For these players, the optimal strategy for them is

$$b_i = \begin{cases} (B-1)mod(v-1)+1 & if\ i\ is\ the\ first\ player \\ Pass & if\ i\ is\ the\ second\ player \end{cases} \quad (18)$$

4.2 The Malicious Strategy

The malicious strategy is the optimal strategy for the malicious players whose spite coefficient is one. These players, instead of maximizing their own profits, prefer to minimize their opponents' benefits. Since in closed markets, letting other opponents shut down may lead to future harvest, it is possible for some mechanisms to become malicious players. The malicious strategy is shown as follows.

$$b_j = \begin{cases} b_i + 1 & if\ b_i < B-(v-1) \\ B & otherwise \end{cases} \quad (19)$$

4.3 The Overbidding Strategy

The overbidding strategy is a simple upgrade of the auction bidding amount and the simplest way to continue the auction. It asks the player to bid 1 and continue with overbidding if his opponent bids.

4.4 Pass

In this situation, it is optimal for the players just simply pass the bid no matter what his opponent bids. Regardless of how he auctioned, he will not be able to win the value. Passing can ensure the player to not lose money, and maximize his utility.

4.5 Summary

In the table below, I summarize the simplification strategies for the alternative auction settings reported by Waniek in 2015 and 2017, where the spite coefficient a_i and a_j of the two players are known, and the steps of the strategy are simplified by similar actions. The first line in each box is the strategy for the first player and the second line is for the second one. The details explanation is omitted due to space constraints.

Table 1. The Strategy in Alternative Settings

THE FIRST PLAYER / THE SECOND PLAYER	$a = 0$	$0 < a < \frac{1}{2}$	$a = \frac{1}{2}$	$\frac{1}{2} < a < 1$	$a < 1$
$a = 0$	Selfish Strategy	Selfish Strategy	Selfish Strategy	Malicious Strategy	Malicious Strategy
	Selfish Strategy	Selfish Strategy	Selfish Strategy	Pass	Pass
$0 < a < \frac{1}{2}$	Selfish Strategy	Selfish Strategy	Selfish Strategy	Malicious Strategy	Malicious Strategy
	Selfish Strategy	Selfish Strategy	Selfish Strategy	Pass	Pass
$a = \frac{1}{2}$	Selfish Strategy	Selfish Strategy	Selfish Strategy	Malicious Strategy	Malicious Strategy
	Selfish Strategy	Selfish Strategy	Selfish Strategy	Pass	Pass
$\frac{1}{2} < a < 1$	Pass	Pass	Pass	Malicious Strategy	Malicious Strategy
	Overbidding Strategy	Overbidding Strategy	Overbidding Strategy	Malicious Strategy	Malicious Strategy
$a < 1$	Pass	Pass	Pass	Malicious Strategy	Malicious Strategy
	Overbidding Strategy	Overbidding Strategy	Overbidding Strategy	Malicious Strategy	Malicious Strategy

If the players know each other's spite coefficient and follow the strategies we mentioned above, we can obtain the results of each situation. I summarize the various situations in the table below. The orange boxes represent the situation where the first player would maximize his utility. And the blue boxes stand for the championship of the second player. The winner is shown in the boxes.

Table 2. The Results in Alternative Settings

THE FIRST PLAYER / THE SECOND PLAYER	$a = 0$	$0 < a < \frac{1}{2}$	$a = \frac{1}{2}$	$\frac{1}{2} < a < 1$	$a < 1$
$a = 0$	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player
$0 < a < \frac{1}{2}$	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player
$a = \frac{1}{2}$	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player	The 1^{st} player
$\frac{1}{2} < a < 1$	The 2^{nd} player	The 2^{nd} player	The 2^{nd} player	The 1^{st} player	The 1^{st} player
$a < 1$	The 2^{nd} player	The 2^{nd} player	The 2^{nd} player	The 1^{st} player	The 1^{st} player

What I want to highlight (*) here is that, when both the players are strongly-spiteful players and malicious players ($\frac{1}{2} < a \leq 1$), they should all follow the malicious strategy. When player i uses the malicious strategy, then a player j will never have the incentive to pass, which can continue the auction and lure the player i. And nobody would pass at any point using the malicious strategy. A dollar auction between two strongly-spiteful players or malicious players ends in a state where both players spend most of their budgets. Surprisingly, when the bid reaches $b - s$, instead of continuing the auction and pressuring the opponent to pay more, both players favor ceasing the auction through biding entire budget B, which resembles to the experiment result in Teger (1980)'s experiment result. Most of the participants said they experienced an autonomous desire to win. The money spent on each subsequent offer could then turn out to be insignificant, making continuing the game a less difficult decision. Although such a strategy is no longer part of the rational auction, it has numerous forms in global escalation, such as the Cold War between the United States and the Soviet Union in the late 1980s.

5. Conclusion

From Table 2, we can clearly observe that, using Waniek's strategy, the player who makes the first bid has a greater probability of winning (maximizing his utility) in most cases. Only in a few cases, when the spite coefficient of the second player is much greater than that of the first player, it is possible for the second one to win. According to the distribution of spitefulness in the crowd, we can see that most people are strongly-spiteful and should follow the malicious strategy. In this situation, two players will likely both spend almost all their budgets, which is consistent with the experimental results we have seen in real life and done in the last century.

What's more, under Waniek's strategy, the more spiteful players are, the higher probability for them to win. This discovery may help the government or the international trade organization to produce some policies to reward enterprises with a low degree of spite coefficient. To my opinion, for economic healthy development, people should pay more attention to maximizing their own profit instead of minimizing their opponents' revenue. To achieve win-win results and maximize each other's benefits is the goal we should pursue, rather than harming others without being beneficial to ourselves. Therefore, relevant organizations need to formulate some policies so that those enterprises defeated only because of the low spite coefficient can get certain rewards and compensation, so as not to make the whole economic environment full of individuals who want to be desperate to

minimize the opponents' profit, resulting in other's property destruction and they would achieve monopolization. This is definitely not a healthy economic environment.

This review also inspired my future work. I hope I can develop an algorithm to judge people's spite coefficient through the analysis of people's strategic behavior. As far as I am concerned, the strategies in the research are not applied to practice, since one major drawback is that the algorithm of judging the players' own spite coefficients had not been taken into account. Although how each other's degree of spitefulness and the best strategy should be related under different circumstances is known, if there is no way to clearly judge people's own spite coefficient, the theoretical research on the best strategy is also futile. Hopefully, by analyzing the action the players take during the auction, it will be able to quantify the players' spitefulness through their choices. The calculated coefficients enable them to accurately execute the optimal strategy of all the spite coefficients to maximize their utilities when the spite coefficients of the players are known.

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