Study of Modern Hiring Strategies Stemmed from the Secretary Problem

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Abstract. The use of the optimal stopping framework is becoming more relied upon in contemporary hiring practices. Various models that are founded on the classical secretary problem emerge to get the best solutions in different circumstances. This review summarizes various articles on the same subject to explore the impact of various variants, including multiple choice, multiple references, parallelization, concurrent employment and multiple employers with strategic competition, on the quality and Hiring efficiency. This paper examines technical contributions, such as Poisson Process of finite arrivals, Primal-Dual linear programming of shared parallel quotas, dynamic programming of temporal coverage, Nash equilibrium to access competitiveness and analytic combinatorics of rank-based selectivity. Key findings indicate that strategic competition re-optimizes the optimal stopping threshold between 37% and 31% and concurrent employment is a mathematical requirement to obtain the same ratios of competition in dynamic environments. The synthesis offers a set of tools to design selective and resilient recruitment systems in the face of various distributional and structural uncertainty.

Keywords: secretary problem, parallelization, Poisson Process, Nash equilibrium.

1. Introduction

In most employment situations, an employer might be required to deal with a large number of applicants and the employer must employ the most desirable applicants so that his firm benefits. One of the simplest methods of doing so is to interview all candidates and compare all candidates to identify the best but it also results in high time and labor expenses. To prevent the huge cost, researchers apply the method of mathematical models to determine the best strategy associated with probability, resulting in the classical secretary problem.

In classical secretary problem, an employer is forced to recruit the most qualified person out of n applicants. The candidates are placed in even random sequence and the employer is only required to make an immediate and final decision of whether or not to hire the candidate. It is common knowledge that the optimum candidate can be recruited with a probability (which is) asymptotically optimal of $1/e$.

In recent years, studies have been conducted on various variants of the secretary problem (e.g. hiring more than one candidate or not a fixed set of hires). Studies also change their objective to discover the unique optimal probability to different angles, such as competitive ratios [1,2]. More variables are taken into consideration to form a more realistic model regarding the hiring strategy, including synergy [3], the impact of teamwork among the candidates, and cost to hire [1].

This literature review is a rigorous synthesis of the papers that fill the gap between theoretical stopping rules and practical recruitment strategies. This literature indicates that the best hiring under the contemporary conditions is to be characterized by: (1) multi-stage, adaptive protocols to share parallel quotas [2,4]; (2) structural parametrization of synergies [3]; (3) the acknowledgement of concurrent employment as a mathematical requirement of cost-efficiency[1]; and (4) the re-tuning of stopping thresholds in the face of competition by The essay is characterized by an analytical and synthesizing task of the main methodologies and findings of the papers.

2. Related Work

Various methodologies of various generalizations and variations of the secretary problem have been examined in this literature review. The multi-choice model is one of the most studied extensions of the secretary problem and it is found in most of the works in the reference of this essay. Researchers are inclined to believe in the role of such variables like contribution of synergy [3] and competition among employers [4].

There is a substantial amount of work on multiple employment quota or parallel arrival queue scenarios. Sun et al. study a parallel model in which interviewers in separate queues have a common quota but do not know what the other one has [2], and demonstrate that adaptive multi-phase protocols are needed to achieve optimality. Besbes et al. examine the effects of gaps in the distribution of candidate quality and a policy [5], called Conservativeness with respect to Gaps (CwG), which can be used to avoid the regret fluctuations that are characteristic of standard heuristics. Whereas Besbes et al. discuss how to overcome environmental uncertainty [5], Turkieltaub Melo deal with strategic uncertainty [4]. Turkeltaub Melo also adds strategic complexity [4], by considering Nash equilibria of competitive hiring and modeling the likelihood of the last hiring k-improving candidate as a Poisson distribution.

The recent models go beyond fixed sequences towards multiple arrivals of candidates. Disser et al. demonstrate the significance of simultaneous hiring and demonstrate that the ability to have overlapping employment contracts is a desirable characteristic to reduce time costs and a fixed competitive ratio of 2.965 with the offline optimal [1]. Also, Li and Yu examine rolling recruitment where arrivals are in batch, and departures are stochastic [6]. Their study reveals that there is a free-riding effect whereby a recruiter can gain an advantage on a larger applicant pool, implying that it is necessary to change the threshold score to enhance the quality of the candidates as the scores of candidates start to diversify.

Feldman and Izsak use a modification of the traditional model to explain the cooperation and synergy in the team [3]. They demonstrate that there exists a drastically reduced learning phase to detect the critical dependencies between synergistic candidates by parameterizing the supermodular degree.

One last literature line considers candidate sequences to be random permutations to examine staff growth. Also using analytic combinatorics, Archibald and Martinez as well as Helmi et al. demonstrate that hiring above the m-th best candidate preserves the overall candidate quality and guarantees the growth in hiring rate to be logarithmic [7,8]. These rank-based models offer the mathematical connection between the discrete hiring actions and long-term staff excellence.

3. Methodology

Considering the various angles of analysis, the probability distribution of the quality of the candidates is quantified and modeled in different ways. Assuming that the arrivals and quality of candidates are independent and discrete, in most of the work of interest, Feldman et al. states that the linear and submodular functions are widely used in the current literature. Feldman et al. adopt an angle of supermodular degree which is less commonly applied, but can offer a realistic model, including the effect of synergy between employees [3].

The time period (a, b) probability of the number of the k-improving candidates is modeled by Turkieltaub et al. as a Poisson distribution with the mean value of $k\,ln(b/a)$ [4], where k is the arbitrary value of the number of candidates that the employer desires to hire and k-improving signifies the candidate being in the top k candidates by the time spot. The probability is given by

$$P[I(a,b)=i]=\frac{(k\ln(b/a))^{i}}{i!}\left(\frac{a}{b}\right)^{k} \tag{1}$$

The work takes into account an infinite arrival model, whereby, the finite arrival model will approach to as the number of the candidates increases. In the conditions of competitive employers,

the Nash equilibrium, which describes a stable state of a system involving multiple participants, is applied to the study of the strategic interactions and how likely the employers are to hire the best candidates. This piece shows the impact of the modelling of the infinite arrivals which can act as a great source of reference to the employers in situations where they are confronted with a large number of applicants and have to compete with others.

Disser et al. use a dynamic programming method, which determines the exact point where the risk of skipping a great candidate is finally outweighed by the risk of settling for a mediocre one [1], to keep the optimal threshold value at time steps and further evaluate the performance of algorithms using Markov Chains. A parallel can be seen in Li et al. [6], dynamic programming of which is to determine a two-threshold policy. The two works present a very clear plan of bringing in candidates on a time basis.

Sun et al. make use of the Primal-Dual Framework [2], where primal means maximizing the total value of candidates hired and dual means minimizing the price assigned to each candidate slot. The methodology consists of defining the probabilities of selection $x_{q,i}^{j|k}$ as primal variables and an almost linear time algorithm to solve the resulting linear programming in $O(nJK^2)$ time in the Q-queue J-choice K-best problem. The global quota is applied to encourage a real-life scenario of parallel interviews which is a method employed to save time cost today.

Archibald, Martinez and Helmi use bivariate generating functions H(z, u) to study the increase of the sets of hiring in permutations [7,8]. By computing the function in terms of u and u= 1, the authors compute the moments of the hiring set size h_n. This method determines a root identity of the set of hiring and Stirling numbers of the first kind $\begin{bmatrix} n \\ k \end{bmatrix}$ left-to-right maxima in permutations, which is counting. It can be used to apply the quasi-powers theorem of Hwang to establish that the size of the hiring set approaches a normal distribution:

$$\frac{h_n - m \ln n}{\sqrt{m \ln n}} \xrightarrow{d} \mathcal{N}(0,1) \tag{2}$$

Archibald et al. and Helmi et al. might be essential to employers whose size of hiring is not fixed, and the findings of convergence to a normal distribution are once more indicative of the importance of Gaussian Distribution in the nature of mathematics [7,8].

In multi-secretary problems whose distributions are continuous, Besbes et al. define the regret by the compensation payment required to make an oracle pick the same option as the online policy [5]. This method converts the discrepancies between an online policy and an offline optimal oracle into a sequence of anticipated compensations. It is demonstrated online that the regret is of the form $\widetilde{\Theta}\left(\boldsymbol{T}^{\mathbf{1/2-1/(2(\beta+1))}}\right)$ when the gap between quality distributions of candidates with zero probability is taken into account, and the regret is of order that. This works will develop a rough mathematical line of approach that may be applied to investigate the regret of online policy which can be extended to issues like network revenue management [5].

To highlight the impact of synergy in employment, Feldman and Izsak add the supermodular degree (d) [3], which quantifies how many other elements can "trigger" an increase in the marginal value of a specific element, to measure how far a valuation function is away to be submodular. The approach uses an estimate-based learning step that is cut short to 1/d of the overall input to learn dependencies between teams before they go out of date. The algorithm presupposes the availability of three different oracles: Value, Independence and Supermodular. The article by Feldman et al. abandons the usage of submodular and linear functions and offers a model that takes into account the impact of teamworking of candidates [3]. The aspect of synergy is a peculiar approach and plays an important role in employee recruitment [9].

4. Results

To make sure that the hiring strategies are optimal, several variables and lenses of the issues have been examined. Others are based on the distinction between the online policy and the offline optimum oracle, whereas others are based on the particular attributes of staff.

The competitive ratios are an indicator of an average performance measure that pits an online algorithm with an offline oracle. The lower bound of the competitive ratios and the optimal value of competitive ratios under various variants are found in the works of Sun et al. [2] and Disser et al. [1]. In Sun et al. [2], competitive ratios quantify the worth of the applicants of the online algorithm and the hiring cost is assessed in Disser et al. [1]. The Feldman et al. algorithm [3] has a uniform matroid constant competitive ratio. The competitive ratios under different variants are shown in Table 1.

Table 1. List of competitive ratios under different variants

Paper	Variants	Result
Sun et al. (2014) [2]	Shared Parallel (2-queue, 2-choice, 2-best)	0.372
Disser et al. (2020) [1]	Temporal (Uniform)	2.965
Disser et al. (2020) [1]	Temporal (Arbitrary)	6.052
Sun et al. (2014) [2]	1-queue K-best	$1 - O\left(\frac{\ln^2 K}{K^2}\right)$ (Lower bound)
Disser et al. (2020) [1]	No-Concurrency	$\Omega(\sqrt{n}/\log n)$ (Lower bound)

Competitive ratio values have been presented, which demonstrate the difference in the performance of the online algorithm in the variants. The publications of Disser et al. and Sun et al. have been found to provide lower bounds of the hiring cost and values of candidates, respectively [1,2]. Based on which, the employers can gain insight into the best and the worst performance of their online algorithm in terms of the quality of the candidates and the cost of hiring. The article by Disser et al. clearly shows the tremendous impact of concurrent employment on the cost of hiring [1].

The Nash Equilibrium (NE) thresholds determine the strategies leading to competitive markets [10]. Turkieltaub et al. discover a distinct NE in the hiring of 2 competing employers with the same pool of candidates of the value $p^* \approx 0.310096$ [4]. The competition compels the employers to recruit 16% prior to the social optimum (e^{-1}) which is 0.37 and the collective likelihood of recruiting high-quality talent is considerably diminished. The article by Turkeiltaub et al. demonstrates the value of collaboration between rivals on the mutual benefits between 2 [4]. The NE across 2 or more competitors can be studied in future studies to provide fewer specific results.

In the article by Helmi et al. [8], the gap between the quality score of the last hired candidate approaches zero in cases where the number of candidates n approaches infinity, and how the quality of hired candidates rises with n in the hiring approach of hiring above the m-th best candidates. There is however, a tradeoff when the m value is changed whereby as m increases there is an increased hiring rate but at the expense of the quality of candidates. It is an intuitive outcome, with the candidates getting hired easier, as the employers are taking into account a bigger pool of quality of candidates, and the quality of hired candidates decreases. The author notes that there is an empty slot of the optimal strategy of keeping the rate and the quality of employees on a balance, which gives future research an opportunity.

The analysis of Li and Yu shows the optimal reward to be convex and submodular in terms of scores of candidate quality. It identifies a free-riding effect where a recruiter can gain when the competitors of an applicant can become stronger because it brings the recruiter nearer to the target of hiring the applicant more effectively [11].

5. Conclusion

Their synthesis suggests that the application of the secretary problem to the contemporary recruitment has undergone a paradigm shift. In modern day research, the classical single best secretary problems have been adjusted at multiple hiring, non-fixed hire set and parallel model. The objective of the investigation has changed to discovering the best probability to such variables as competitive ratios or size distribution of hiring. Methodologies have also many variations, a few of which are revolutionary methodologies, such as the use of supermodular degree in place of the submodular functions and multidisciplinary approach of applying game theory. In some cases, optimal conditions are found, for example concurrent employment in parallel quotas and large number of candidates in most case. However, in other instances, the best strategies remain open questions to be answered in the future. This branching of variation in works and the ambiguities of best strategies show the future possibilities of the branches of the classical secretary problem, and these papers offer a mathematical toolkit to design recruitment systems which are not just efficient but robust to the uncertainties of new digital labor markets.

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